A matrix with no real eigenvalues

The standard geometric interpretation of the linear transformation with matrix

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

is that it represents a counterclockwise rotation by 90 degrees about the origin. From this viewpoint it is clear that the matrix has no real eigenvalues or eigenvectors, for it takes a vector (x, y) to its image under this rotation. The rotated vector is never a scalar multiple of the original vector provided the latter is nonzero, and hence there cannot be any real eigenvectors or eigenvalues.