We have noted that the derivation of an explicit formula for a determinant function does not immediately imply that there is a function with all the right properties. This tricky logical point is related to material on ***extraneous roots*** that one sometimes finds in elementary algebra courses. Here is a quick review of the underlying ideas. Suppose that we want to solve an equation like

$$x - 3 = \sqrt{30 - 2x}$$

The standard way to attack this type of problem is to eliminate the radical by squaring both sides and solving for x:

$$(x-3)^2 = \left(\sqrt{30-2x}\right)^2$$
$$x^2 - 6x + 9 = 30 - 2x$$
$$x^2 - 4x - 21 = 0$$
$$(x-7)(x+3) = 0$$
$$x = 7; x = -3$$

(Source: http://regentsprep.org/Regents/mathb/7D3/radlesson.htm)

This tells us that **the only possible solutions are given by the two values above, but *it does not guarantee that either is a solution.*** The reason for this is that the first step, in which we square both sides, shows that the first equation implies the second, but it does **not** follow that the second implies the first; for example, even though the squares of 2 and -2 are equal, these two numbers are clearly **not** the same. In order to complete the solution of the problem, we need to go back and determine which, if any, of these two possible solutions will work. It turns out that $x = 7$ is a solution, but on the other hand $x = -3$ is not (and hence it is an extraneous root).

The online site **http://www.jimloy.com/algebra/square.htm** has further examples of this type.