ADDITIONAL EXERCISES FOR MATHEMATICS 132

WINTER 2004

Many of these problems are taken from the current text for another departmental linear algebra course (Mathematics 23), and the references indicate sections and pages of that text:

> R. Larson, B. H. Edwards and D. C. Falvo, *Elementary Linear Algebra* (Fifth Edition), Houghton Mifflin, Boston and New York, 2004. ISBN: 0–618–33567–6.

This book is on two hour reserve in the Science Library.

NOTE. The only purpose of these recommended exercises is to provide students with opportunities for extra practice in working certain basic types of problems. There is no obligation to work any of these exercises, so students should work as many or as few as they wish.

I. Eigenvalues and eigenvectors

I.1 : Basic definitions

Larson, Edwards and Falvo, § 3.4, p. 152: 5–15 odd [but ignore the comment about using computers]

Larson, Edwards and Falvo, § 7.1, p. 421: 13–23 odd

I.2 : Diagonalization

Larson, Edwards and Falvo, § 7.2, p. 432: 13–29 odd

I.3 : Differential and difference equations

Larson, Edwards and Falvo, § 7.4, p. 459: 15–19 odd

II. Perpendicularity (Orthogonality)

II.A : Review topics

Larson, Edwards and Falvo, § 5.2, p. 295: 51

II.1 : Orthogonal bases

Larson, Edwards and Falvo, § 5.3, p. 310: 13–31 odd, 39, 41

II.2 : Orthogonal projections and adjoints

Larson, Edwards and Falvo, § 5.4, p. 325: 11, 13

II.3 : Orthogonal matrices

Larson, Edwards and Falvo, § 7.3, p. 444: 13–17 odd

III. Change of bases

III.1 : Similarity of matrices

Larson, Edwards and Falvo, § 6.4, p. 395: 3–7 odd

III.2 : Invariants of similarity

Larson, Edwards and Falvo, § 7.2, p. 432: 45

IV. Complex linear algebra

Solutions to the exercises for IV.1–IV.3 may be found in the file `linalgexercises2solns.pdf`.

IV.1 : Complex numbers

Additional exercises

1. Find the products of the following complex numbers:
(a) $(5 - 5\,i)(1 + 3\,i)$
(b) $(\sqrt{7} + i)(\sqrt{7} - i)$
(c) $(a + b\,i)^2$
(d) $(1 + i)^3$
(e) $(a + b\,i)^3$

2. Find the absolute value (or modulus) of $z\,w$, where $z = 2 + i$ and $w = -3 + 2\,i$.

3. Simplify the following expression:

$$\frac{(2 + 3\,i)(3 - i)}{4 - 2\,i}$$

4. Perform the indicated operations and leave the result in the polar form $z = r(\cos\theta + i\,\sin\theta$:
(a)

$$\frac{2[\cos(2\pi/3) + i\,\sin(2\pi/3)]}{4[\cos(2\pi/9) + i\,\sin(2\pi/9)]}$$

(b)

$$\frac{12[\cos(\pi/3) + i\,\sin(\pi/3)]}{3[\cos(\pi/6) + i\,\sin(\pi/6)]}$$

5. Use de Moivre's Theorem to find the indicated powers of the given complex numbers, and express the results in standard form:
(a) $(1 + i)^4$
(b) $(-1 + i)^{10}$
(c) $(1 - \sqrt{3}\,i)^3$

6. Use de Moivre's Theorem to find the cube roots of 8 and 27, and express these in standard form.

IV.2 : Complex matrices

Additional exercises

1. Perform the indicated matrix operations using the following comples matrices:

$$A = \begin{pmatrix} 1+i & i \\ 2-2i & -3i \end{pmatrix} \qquad \text{and} \qquad B = \begin{pmatrix} 1-i & 3i \\ -3i & -i \end{pmatrix}$$

(a) $A+B$
(b) $2A$
(c) $2iA$
(d) $\det(A+B)$
(e) $5AB$

2. Determine whether the following complex matrices are invertible, and find the inverse for each matrix that is invertible.

(a)

$$\begin{pmatrix} 6 & 3i \\ 2-i & i \end{pmatrix}$$

(b)

$$\begin{pmatrix} 1-i & 2 \\ 1 & 1+i \end{pmatrix}$$

3. For which complex numbers z is the matrix

$$\begin{pmatrix} 5 & z \\ 3i & 2-i \end{pmatrix}$$

NOT invertible? [*Hint:* Set $\det A = 0$ and solve for z.]

$$\frac{(2+3i)(3-i)}{4-2i}$$

4. For each of the folowing lists of n vectors in $\mathbb{C}^n$, determine whether it is a basis for $\mathbb{C}^n$:
(a) $\mathcal{A} = \{\,(1,i),\ (i,-1)\,\}$
(b) $\mathcal{A} = \{\,(i,0,0),\ (0,i,0),\ (0,0,1)\,\}$

5. Find the lengths of each of the following complex vectors:
(a) $(1, 2+i, -i)$
(b) $(2, -1+i, 2-i, 4i)$

6. Find the distance between the complex vectors $(1, 2i,\ 3i)$ and $(0,1,0)$.

7. Determine whether the folowingsets of vectors are linearly independent or linarly dependent:
(a) $\mathcal{A} = \{\,(1,i),\ (i,-1)\,\}$
(b) $\mathcal{A} = \{\,(1+i, i, 1+i),\ (0,-i,i),\ (0,0,1)\,\}$

8. If A and $\mathbf{w}$ are the 3×2 and 3×1 matrices given below, find all 2×1 matrices $\mathbf{x}$ such that $A\mathbf{x} = \mathbf{w}$:

$$A = \begin{pmatrix} 1 & 0 \\ i & 0 \\ i & i \end{pmatrix}, \qquad \mathbf{w} = \begin{pmatrix} 2 \\ 2i \\ 3i \end{pmatrix}$$

9. Determine the conjugate transpose matrix A^* if A is the following matrix:

(a)

$$\begin{pmatrix} 1 & -i \\ 2 & 3i \end{pmatrix}$$

(b)

$$\begin{pmatrix} 0 & 5+i & \sqrt(2)\, i \\ 5-i & 6 & 4 \\ -\sqrt{2}\, i & 4 & 3 \end{pmatrix}$$

10. Determine whether each of the following matrices is Hermitian:

(a)

$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

(b)

$$\begin{pmatrix} 0 & 2+i & 1 \\ 2-i & i & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

IV.3 : Complex eigenvalues and eigenvectors

Additional exercise

1. Find the eigenvalues and eigenvectors for the following matrices:

(a)

$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

(b)

$$\begin{pmatrix} 2 & -i/\sqrt{2} & i/\sqrt{2} \\ i/\sqrt{2} & 2 & 0 \\ -i/\sqrt{2} & 0 & 2 \end{pmatrix}$$

IV.4 : Jordan form

IV.5 : Differential equations revisited

Additional exercise

1. In the first example on page 450 of Larson—Edwards—Falvo there is an assertion about solutions to a specific system of linear equations. Use the methods developed in the course notes to prove that the functions y_1 and y_2 presented in this example are in fact the solutions to the given system of differential equations.

V. Quadratic forms

Solutions to the exercises for V.3 may be found in the file `linalgexercises2solns.pdf`.

V.1 : Diagonalization of quadratic forms

Larson, Edwards and Falvo, § 7.3, p. 444: 7, 11, 21–25 odd

V.2 : Classification of quadrics

Larson, Edwards and Falvo, § 7.4, p. 459: 33–45 odd

1.**

V.3 : Classification of critical points

Additional exercise

1. Find the critical points of the functions listed below and determine use the second derivative test to determine if they are relative maxima, relative minima, saddle points or if the test fails:

(a) $x^2 + y^2 + 6\,x + -4\,y$.
(b) $x^2 + y^2 + 3\,x\,y$.
(c) $\exp(1 + x^2 - y^2)$.
(d) $x^3 - 3\,x\,y + y^3$.
(e) $x^2 + y^2 + x^2\,y + 4$.
(f) $e^x\,\cos y$.
(g) $x\,\sin y$