SOLUTIONS TO SELECTED ADDITIONAL EXERCISES FOR MATHEMATICS 132 FOR UNITS IV AND V FROM

`linalgexercises2.pdf`

Winter 2004

IV. Complex linear algebra

IV.1 : Complex numbers

Additional exercises

1. (a) $20 + 10\,i$ (b) 8 (c) $(a^2 - b^2) + 2\,a\,b\,i$ (d) $-2 + 2\,i$ (e) $(a^3 - 3\,a\,b^2) + (3\,a^2\,b - b^3)\,i$

2. 65

3. $\frac{13}{10} + \frac{9}{10}\,i$

4. (a) $\frac{1}{2} \cdot (\cos(4\pi/9) + i\,\sin(4\pi/9))$

5. (a) -4 (b) $-32\,i$ (c) -8

6. The cube roots of 8 are 2 and

$$2\left(\cos(2\pi/3) \ \pm\ i\,\sin(2\pi/3)\right)$$

and the cube roots of 27 are 3 and

$$3\left(\cos(2\pi/3) \ \pm\ i\,\sin(2\pi/3)\right) .$$

Note that

$$\cos(2\pi/3) \ - \ i\,\sin(2\pi/3) \quad = \quad \cos(4\pi/3) \ + \ i\,\sin(4\pi/3) .$$

IV.2 : Complex matrices

1.

(a)

$$\begin{pmatrix} 2 & 1 + 3\,i \\ -1 - 2\,i & -4\,i \end{pmatrix}$$

(b)

$$\begin{pmatrix} 2 + 2\,i & 2 \\ 4 - 4\,i & -6\,i \end{pmatrix}$$

(c)

$$\begin{pmatrix} -2+2\,i & 2\,i \\ 4+4\,i & 6 \end{pmatrix}$$

(d)

$$-5-3\,i$$

(e)

$$\begin{pmatrix} -5 & -15+10\,i \\ 25\,i & 15+30\,i \end{pmatrix}$$

2. (a) Not invertible. (b) The inverse matrix is equal to

$$-\frac{1}{3}\cdot\begin{pmatrix} i & -3\,i \\ -2+i & 6 \end{pmatrix}\,.$$

3. The matrix is not invertible if and only if $z=-\frac{5}{3}-\frac{10}{3}\,i$.

4. (a) Not a basis. (b) Is a basis.

5. (a) $\sqrt{7}$ (b) $\sqrt{17}$

6. $\sqrt{15}$

7. (a) Linearly dependent. (b) Linearly independent.

8. $\mathbf{x}$ is equal to the transpose of $(2 \quad 1)$.

9.
(a)

$$\begin{pmatrix} 1 & 2 \\ i & -3\,i \end{pmatrix}$$

(b)

$$\begin{pmatrix} 0 & 5+i & \sqrt{2}\,i \\ 5-i & 6 & 4 \\ -\sqrt{2}\,i & 4 & 3 \end{pmatrix}$$

10. (a) Hermitian. (b) Not Hermitian; one of its diagonal entries is imaginary, and the diagonal entries of a Hermitian matrix are real.

IV.3 : Complex eigenvalues and eigenvectors

1. (a) The eigenvalues are ± 1 and associated eigenvectors are given by $\mathbf{v}_1=(1,-i)$ and $\mathbf{v}_{-1}=(1,i)$.

(b) The eigenvalues are 1, 2 and 3, and associated eigenvectors are given by $\mathbf{v}_1=(\sqrt{2},\,-i,\,i)$, $\mathbf{v}_2=(0,1,1)$ and $\mathbf{v}_3=(\sqrt{2},\,i,\,-i)$.

V. Quadratic forms

V.3 : Classification of critical points

1. (a) $(-3, 2)$ is a relative minimum. (b) $(-18/5,\ -11/15)$ is a relative minimum. (c) $(0, 0)$ is a saddle point. (d) $(0, 0)$ is a saddle point and $(1, 1)$ is a relative minimum. (e) $(0, 0)$ is a relative minimum and $(\pm\sqrt{2},\ 1)$ are saddle points. (f) No critical points. (g) $(0, n\,\pi)$ is a saddle point, where n is an integer.