Math 132
Winter 2004
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REVIEW RECOMMENDATIONS — JANUARY 26, 2004

The following are some recommendations for review in connection with the upcoming first midterm examination on Monday, February 2.

The coverage of the examination will correspond to everything in the online course notes `linalgnotes.pdf` through the end of Section II.2. There will be some minor revisions to the latter that will be complete by the beginning of class on Wednesday, January 28. The corresponding material in the text is contained in Chapter 5 and Sections 1, 2 and 4 of Chapter 6.

Most of the examination at least 65 per cent) will consist of exercises from the assignment document `linalgexercises.pdf` in the course directory. The document `linalgexercises2.pdf` is only meant as extra practice for anyone who feels the need to do more exercises in order to sharpen his or her skills. Students are responsible for knowing how to do the exercises not marked by one or two stars in `linalgexercises.pdf`.

Here are some further suggestions.

Working examples. Most of the points on the exam will be for problems of this type. The main types of problems involve finding eigenvalues and eigenvectors for specific matrices, computing expressions like $A^k\,\mathbf{x}$ when A is diagonalizable using a basis of eigenvectors for A, using the formula in the Gram-Schmidt process to find orthonormal bases and perpendicular projections onto subspaces, and finding the matrix representing a perpendicular projection onto a subspace.

Definitions and key results. Important points here include definitions of eigenvalues and eigenvectors, the characteristic polynomial and its use in finding eigenvalues, diagonalizability, some different types of matrices that are not diagonalizable (no real roots, all real roots but still no basis of eigenvectors), linear independence of eigenvectors with distinct eigenvalues, inner product spaces, linear independence of sets of nonzero pairwise orthogonal vectors, orthonormal bases, orthogonal complements, the Gram-Schmidt process, the formula for coordinates of a vector with respect to an orthonormal basis in terms of the inner product, perpendicular projections onto subspaces, the Least Squares Minimization Property, adjoint linear transformations, self-adjoint transformations, and the relation of the latter to the transposition operation on matrices.

Simple proofs and derivations. The shorter items in the assigned exercises are worth reviewing in detail (if a solution takes more than about a quarter of a page, it is safe to assume it will not appear on the examination). Here are some further suggestions for things that might appear. On the subject of eigenvalues and eigenvectors, there are the proof that λ is an eigenvalue if and only if it is a root of the characteristic polynomial, the proof that a 3×3 matrix with real entries always has a real eigenvalue, the linear independence of two or three eigenvectors associated to distinct eigenvalues, diagonalizability of $n \times n$ matrices with n distinct eigenvalues. On the subject of perpendicularly, the possibilities include the existence of orthonormal bases (from the Gram-Schmidt process), the formula for coordinates of a vector with respect to an orthonormal basis in terms of the inner product, and expressibility of a vector as a sum of one vector in a given subspace and another in its orthogonal complement. No derivations involving adjoint transformations will be included except possibly those in the assigned exercises.

Partial credit

The grading for each problem will be broken down into individual steps, each of which is worth a relatively small number of points if completed successfully. Additional partial credit may be given for steps that are not successfully completed but for which the attempt indicates sufficient insight or understanding.

Suggestions regarding the third assignment

It might be useful to look at some problems that are closely related to the ones that have been assigned but have answers in the back of the text. Here is a copy of the textbook assignments together with similar problems from the text for which answers are given in the book:

- Exercises 2, 6, 14a and 34 beginning on p. 336 of F&B. *Exercises* 1, 5, 13a *and* 33 *or* 35 *are similar*.
- Exercise 2 beginning on p. 347 of F&B. *Exercises* 1 *and* 3 *are similar*.
- Exercise 26 beginning on p. 368 of F&B. *Exercises* 25 *and* 27 *are similar*.
- Additional Exercises 3ab and 4b for Section II.2

Here are some hints for the Additional Exercises. For 3a, note that $(T\,S)^* = S^*\,T^*$ and $I^* = I$, and similarly if S and T are reversed. For 3b, let A be the matrix representing T with respect to particular ordered orthonormal bases for V and W. What is the matrix for T^*? Why is the rank of T equal to the column rank of A? How are the column ranks of A and its transpose related? For 4b, find a relation between the rank and nullity of T^*.

Related documents in the course directory

In addition to the ones mentioned above, there are documents named `solutions1.pdf` and `solutions2.pdf` that have solutions to some of the exercises from the first assignment.