Math 132
Winter 2004
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UPDATED GENERAL INFORMATION — MARCH 11, 2004

In response to several student queries about grade calculation, the course grading policy will be adjusted as follows: If the weighted average grade on the two midterm examination and the final examinations is higher than the grade on the homework, then the final grade will be determined using only the weighted average on the examinations.

Further suggestions regarding the final examination

These previous suggestions regarding the first and second midterm examinations also apply to the final examination.

There may be some problems on the examination of a mixed conceptual and computational nature. The point of such problems would be to demonstrate that certain statements do not necessarily hold for matrices. Two easy examples of this type would be that one does not necessarily have $AB = BA$ for $n \times n$ matrices or that the sum of two invertible matrices is not necessarily invertible. The latter really involve the prerequisite to this course, and some examples more directly related to this course would be to show that the eigenvalues of a sum matrix $A + B$ do not necessarily have the form $\alpha + \beta$ where α is an eigenvalue for A and β is an eigenvalue for B as well as a similar statement for products. For such an example, you might be given two specific matrices A and B and asked to show that the eigenvalues of the sum or product matrix are not the sums of products of the eigenvalues of A and B.

Several problems will have definitions and hints included. Note that some hints might not give the best or only way to work the problem in question, and any mathematically correct method may be used to obtain answers (but of course the reasons for the answers need to be clear from what you write down).

Problems on finding eigenvectors and eigenvalues can be expected, but in some problems it is likely that the objective will be to compute eigenvectors given the eigenvalues while in others the objective will be to compute the eigenvalues and nothing more. Similarly, problems involving the Gram-Schmidt process or perpendicular projections can be expected. Something involving matrix operations from the prerequisite course but with complex scalars is another thing that is likely to appear. As before, something about finding Jordan forms may be on the examination, but there will not be any examples where you are asked to find the Jordan form without information on some of the preliminary steps. On the topic of linear algebra over the complex numbers, there might be something on complex eigenvectors and eigenvalues for real matrices; recall that if a real matrix has nonreal eigenvalues and eigenvectors, then the latter come in complex conjugate pairs (this is the only part of IV.5 that was actually covered in class). Special types of diagonalizable complex matrices may appear on the examination in some form.

Regarding the last unit of the course, the most important concept is the description of quadratic forms in terms of real symmetric matrices and the diagonalizability of such forms by an orthogonal change of variables. Note that one only needs to find the eigenvalues of the symmetric in order to write down the diagonalized version of the quadratic form. For the applications of quadratic forms to recognizing conics or quadrics, or more generally hyperquadrics in $\mathbf{R}^n$, and also to the second derivative tests for extrema of functions of several variables, it is often enough to know how many eigenvalues are positive, negative or zero. Something involving the principal minors tests for all positive and all negative eigenvalues is likely to appear on the examination. In Wednesday's lecture there were some comments about finding the numbers of positive and negative eigenvalues for real symmetric 3×3 matrices that did not have all positive or negative eigenvalues but had nonzero determinants. You should understand what happens in such conditions (both for positive and negative determinants) and the implications for describing the type of a quadric surface defined by an equation ${}^{\mathbf{T}}\mathbf{x}\, A\, \mathbf{x} = 1$ where A is an invertible symmetric matrix. Similarly, you should understand what sort of quadrics are possible if A is not invertible and 0 is an eigenvalue with algebraic multiplicity one.