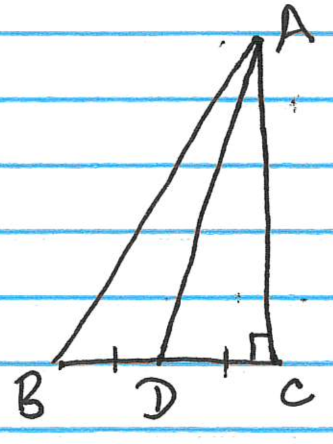
SOLUTIONS FOR aabUpdate08f18.pdf

1.



We have

$$\tan |\measuredangle BAC| = \frac{|BC|}{|AC|}$$

$$\tan |\measuredangle DAC| = \frac{|DC|}{|AC|}$$

If $[AD$ bisects $\measuredangle BAC$, then these imply that

$$\tan \tfrac{1}{2}|\measuredangle BAC| = \frac{|DC|}{|AC|} = \frac{1}{2}\,\frac{|BC|}{|AC|} = \tfrac{1}{2}\tan|\measuredangle BAC|.$$

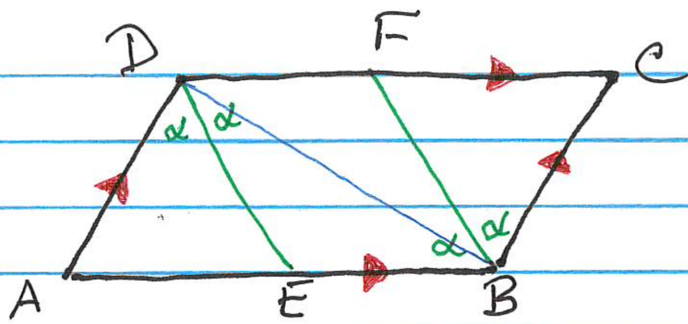
Now consider the equation $\tan\frac{1}{2}\theta = \frac{1}{2}\tan\theta$.
For what values of θ does this hold? The claim is that there are no such values, and in fact
$f(\theta) = \frac{1}{2}\tan\theta - \tan\frac{1}{2}\theta > 0$ for $0 < \theta < \frac{\pi}{2}$.

One way to prove this is to show that $f'(\theta) > 0$ in this range; since $f(0) = 0$, this means $f(\theta) > 0$ as well. But

$$f'(\theta) = \frac{1}{2}\sec^2\theta - \frac{1}{2}\sec^2\tfrac{1}{2}\theta$$

and this is positive because $\sec\theta$ is strictly increasing for $0 < \theta < \frac{\pi}{2}$ (its reciprocal, $\cos\theta$ is strictly decreasing and positive). ■

2.



We know that $|\measuredangle ADC| = |\measuredangle ABC|$, and since $[DE$ and $[BF$ are bisectors we have

$$|\measuredangle ADE| = |\measuredangle DEC| = |\measuredangle ABF| = |\measuredangle BFC| .$$

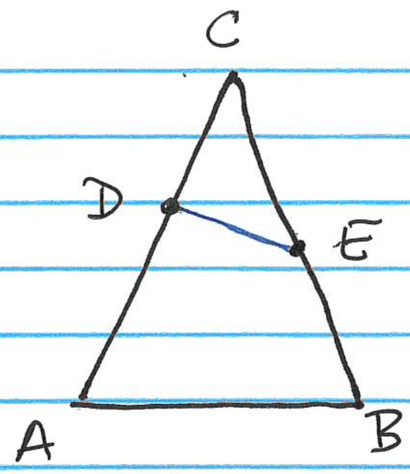
Since A, B, C, D form the vertices of a parallelogram (which is a convex quadrilateral), it follows that C and F are on one side of BD while A and E are on the other [(AC) meets (BD) at some point X, and $A*E*B$ and $C*F*D$ are given].

Since a diagonal splits a $\square$ into two congruent $\triangle$s we have $|\measuredangle ADB| = |\measuredangle DBC|$. Now E and F lie in the interiors of $\measuredangle ADB$ and $\measuredangle DBC$ respectively, we have

$$|\measuredangle BDE| = |\measuredangle BDA| - \alpha = |\measuredangle CDB| - \alpha = |\measuredangle FBD| .$$

Therefore the transversal BD determines alternate interior angles for DE and FB with equal measures, and consequently $DE \parallel FB$. ■

3.



By the Isosceles Δ Thm., $|\measuredangle CAB| = |\measuredangle CBA|$

In ΔCDE we have

$$|CD| = |AC| - |AD| < |BC| - |BE| = |CE|.$$

because $|AD| > |BE|$, $A * D * C$ and $B * E * C$

Since the larger angle is opposite the longer side, we have $|\measuredangle CDE| > |\measuredangle CED|$. By the supplement postulate, we then have

$$|\measuredangle ADE| = 180 - |\measuredangle CDE| < 180 - |\measuredangle CED| = |\measuredangle BED|$$

which was the objective of the problem. ∎