

UPDATED GENERAL INFORMATION — NOVEMBER 20, 2018

The third quiz will take place on **Tuesday, November 27, 2018**. It will cover material from Sections III.3–III.5. The problems will involve algebraic techniques and solving equations for numerical values, finding one ratio in terms of others, or showing inequalities of lengths or angle measures. In addition to problems in the assignments and `quiz3.pdf`, working the problems below is strongly recommended.

1. Suppose that $\triangle ABC$ is a right triangle with a right angle at B , with the lengths of the sides given by $|AB| = 7$, $|BC| = 24$ and $|AC| = 25$. Let $X, Y \in (AC)$ be such that $[BX]$ is an altitude from B to $[AC]$ and $[BY]$ bisects $\angle ABC$. Find the lengths AX and AY .
2. Suppose we are given isosceles triangle $\triangle ABC$ in the coordinate plane, where $A = (0, 3)$, $B = (-1, 0)$ and $C = (1, 0)$; denote $(0, 0)$ by Q . Let $D \in (AC)$ be the foot of the perpendicular from Q to AC , and let $E \in (AC)$ be chosen such that $[BE]$ bisects $\angle ABC$. Find the lengths $|AE|$ and $|AD|$.
3. Suppose that $\triangle ABC$ is a right triangle with a right angle at B , with the measures of the remaining angles given by $|\angle BAC| = 15^\circ$ and $|\angle ACB| = 75^\circ$. If D is the midpoint of $[AC]$, find $|\angle ADB|$. [*Hint:* E be the midpoint of $[BC]$, and let F be the midpoint of $[AB]$. Find four triangles whose vertices lie in $\{A, B, C, D, E, F\}$ and are similar to $\triangle ABC$ with the same ratio of similitude.]
4. Suppose we are given (convex) trapezoid $ABCD$ with $AB, CD \perp AD$. Show that if $|DC| < |AB|$, then $|\angle DCB| > 90^\circ > |\angle ABC|$. [*Hint:* Suppose that $E \in (DC)$ is such that $|DE| = |AB|$, show that $D * C * E$ holds and A, B, C, D (in that order) are the vertices of a rectangle. Now apply the Exterior Angle Theorem to $\triangle CEB$.]
5. Suppose that A, B, C, D (in that order) are the vertices of a rhombus (a parallelogram with all sides of equal length), and let X be the point where the diagonals meet. Suppose that we have $|AX| = |BX| = |CX| = |DX|$. Is the rhombus a square? Prove this or give a counterexample.