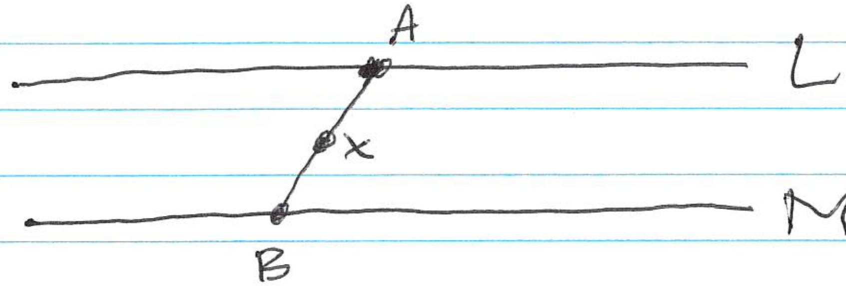
SOLUTIONS FOR THE THIRD GROUP OF PROBLEMS

1.



We know $X \notin L$ and $X \notin M$, so one of $X*A*B$, $A*X*B$, $A*B*X$ is true. But $X*A*B$ implies X and B are on opposite sides of L, while $A*B*X$ implies X and A are on opposite sides of M. By assumption neither of these is true, so the only remaining possibility is $A*X*B$. ■

2. Since $AB \perp BC$, $CD \perp BC$ and $EF \perp BE (= BC)$, it follows that each of the lines AB, CD, EF are parallel to the others. Now $B*C*E \Rightarrow$ B and E are on opposite sides of CD, so AB and EF are contained in opposite sides. Thus (AF) meets CD in some point X.

If A, D, F are collinear, then $X \in AF \cap CD$ $\Rightarrow X = D$. Hence $|\measuredangle ADC| + |\measuredangle CDF| = 180°$. The assumptions imply $|\measuredangle ADC| = |\measuredangle CDF|$ (verify this; the top angles of a S. quad have equal measure & one can split both quads into pairs of $\cong$ triangles). Therefore $|\measuredangle ADC| = |\measuredangle CDF| = 90°$ and the top angles are all right angles, so that $ABCD$ & $DCEF$ are rectangles. ■

Conversely, ~~if~~ assume both are rectangles, ~~then~~ As before, A + F lie on opp sides of CD and we have $X \in (AF) \cap CD$. Let $G \in AD$ so that $A * D * G$ and $|DG| = |DF|$. Then the supplement property implies $|\measuredangle CDG| = 180 - |\measuredangle CDA| = 90$, so the protractor property implies $[DF = [DG$. Finally, $|DG| = |DF|$ now implies $F = G$. Hence A, D and $F (= G)$ are collinear. ■

3. We have $\triangle ABC \sim \triangle ADE$ so that

$$\frac{|AC|}{|AE|} = \frac{|AB|}{|AD|}, = \frac{|BC|}{|DE|},$$ or

$$\frac{15}{15+x} = \frac{12}{30}.$$ Solve for x. $\blacksquare$

$$\left(= \frac{2}{5}\right)$$

4. $\measuredangle DFA$ and $\measuredangle EFC$ are vertical angles, so $|\measuredangle DFA| = \cancel{|\measuredangle CFE|}\ |\measuredangle CFE|$

$\measuredangle ADF$ and $\measuredangle CEF$ are alternate interior angles, so $|\measuredangle ADF| = |\measuredangle CEF|$.

By AA similarity, $\triangle DFA \sim \triangle CFE$

Hence $\frac{|AF|}{|FC|} = \frac{|AD|}{|CE|}$. Substituting, get

(Since $|CE| = 12 = 18 - 6 = |CB| - |EB|$)

$$\frac{3}{2} = \frac{18}{12} = \frac{24}{x}.$$ Solve for x. $\blacksquare$