Cross – references from Greenberg

Unit I (Topics from linear algebra)

Comments.

Greenberg's book approaches the subject from a much different angle, and because of this there is very little in that book which corresponds to Unit **I** of the notes. However, the online reference

http://math.ucr.edu/~res/progeom/pgnotes01.pdf

summarizes much of the material in the first three sections of the notes, and as indicated in the latter pages 13 – 30 of

http://math.ucr.edu/~res/progeom/pgnotes02.pdf

develop the basic material in Section **I.4** at a slightly higher level. A compressed review of the linear algebra needed for the notes (again at a more advanced level) is given in an appendix to the online notes cited above:

http://math.ucr.edu/~res/progeom/pgnotesappa.pdf

Unit II (Vector algebra and Euclidean geometry)

General convention. Unless indicated otherwise, the background readings for a section of the course will begin with a subheading on the initial page and will end just before a subheading on the final page.

II.1 : Approaches to Euclidean geometry

Supplementary background readings.

Greenberg : pp. 34 – 38, 69 – 81

Comments.

The first passage discusses the synthetic and analytic approaches from a historical viewpoint, and the second discusses the incidence axioms. Statements of the axioms along with a few simple consequences appear in the first subheading (pp. 69 – 71), models like the finite planes of the notes are considered in the second subheading (pp. 72 – 76), and the final two subheadings (pp. 76 – 81) discuss issues of logical consistency and isomorphic (***i.e.***, mathematically equivalent) models from a somewhat more advanced point of view than the course notes.

II.2 : Synthetic axioms of order and separation

Supplementary background readings.

Greenberg : pp. 25 – 28, 103 – 115 (through the statement of Proposition 6.3)

Comments.

The first passages discusses the pluses and minuses of using drawings as aids to understanding geometrical proofs, and it includes a version of the fallacious argument which "proves" that all triangles are isosceles. In the second passage, the first subheading discusses some of the logical "bugs" in the Elements and some remarks about Hilbert, and the second subheading (as restricted above) discusses the mathematical subject matter in this section of the notes.

II.3 : Measurement axioms

Supplementary background readings.

Greenberg : pp. 115 – 121 (up to Congruence Axiom 6), 124 – 128 (through Proposition 3.21)

Comments.

The first passage contains versions of the axioms for linear and angular measurement, and the second contains some elementary consequences of the axioms.

II.4 : Congruence, superposition and isometries

Supplementary background readings.

Greenberg : pp. 121 – 129 (all parts of the subheading not covered previously) , 397 – 402, 411 – 421

Comments.

The passages on pp. 121 – 129 contain the congruence axioms for triangles and some basic consequences of them. The file

http://math.ucr.edu/~res/trianglecongruence.pdf

explains how the three triangle congruence axioms in our treatment can be reduced to a single **SAS** axiom as in Greenberg. On pp. 397 – 402 the viewpoint of Felix Klein on geometry and transformations preserving geometrical structure is discussed; this viewpoint is not mentioned in the notes, but the notion of structure preserving geometric transformations is studied at some length in this section of the notes. Important examples of measurement preserving transformations are analyzed on pp. 411 – 421, and this is continued on pp. 426 – 431. The relations between the notions of congruence, geometric transformations, and the symmetries of a geometrical figure are discussed on pp. 444 – 451.

II.5 : Euclidean parallelism

Supplementary background readings.

Greenberg : pp. 138 – 139 (up to the first full paragraph), 81 – 85 (from the second definition up to the first full paragraph)

Comments.

The first passage discusses Playfair's Postulate, and the second discusses mathematical systems which satisfy only the incidence axioms and Playfair's Postulate. Also, on pp. 424 – 425 there is a discussion of an associated class of geometric transformations, but it uses concepts not introduced in the units of the notes which are covered in the course.

Unit III (Basic Euclidean concepts and theorems)

III.1 : Perpendicular lines and planes

Comments.

The readings for the next section also discuss some of the results from this section.

III.2 : Basic theorems on triangles

Supplementary background readings.

Greenberg : pp. 161 – 176, 403 – 408

Comments.

The proofs in the first passage are entirely synthetic, and those in the first subheading of the latter do not use Playfair's Postulate. In contrast, the proofs in the second subheading of the first passage do use that assumption. The second passage indicates how one can use geometric transformations to prove basic results in geometry.

III.3 : Convex polygons

Supplementary background readings.

Greenberg : p. 187 (the definition and footnote)

Comments.

The basic concepts and results in this section do not receive much coverage in Greenberg.

III.4 : Concurrence theorems

Supplementary background readings.

Greenberg : pp. 403 – 408 (up to and including Figure 9.8)

Comments.

This passage discusses proofs of several results related to the concurrence theorems using geometric transformations.

III.5 : Similarity

Supplementary background readings.

Greenberg : pp. 408 – 410

Comments.

This passage discusses similarity transformations in terms that differ considerably from the notes; at points where various sorts of planes are discussed, one can simply interpret phrases like Hilbert plane to mean the ordinary Euclidean plane as in the class notes, and comments regarding non – Archimdean systems can be ignored.

III.6 : Circles and constructions

Supplementary background readings.

Greenberg : pp. 29 – 34,129 – 138, 365 – 369

Comments.

The first passage discusses constructions with (unmarked) straightedges and compasses. The second passage gives geometric interpretations of some basic properties of the real number system, and particularly their uses to find intersection points of a circle with a line or another circle. The discussion of the Dedekind completeness axiom beginning on page 135 involves concepts that are relatively sophisticated and may be troublesome. As suggested on page 135 of Greeenberg, for nearly all purposes in this course it will suffice to think of the real numbers as a system such that **(1)** every nonnegative number has a nonnegative square root, **(2)** between any two real numbers there is a rational number. The third passage contains some fairly challenging exercises in advanced Euclidean geometry.

III.7 : Areas and volumes

Comments.

This material is not covered in Greenberg.