**NAME:** ______________________________

# Mathematics 133, Fall 2018, Examination 1

*Work all questions, and unless indicated otherwise give reasons for your answers. The point values for individual problems are indicated in brackets.*

| # | SCORE |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| TOTAL | |

1. [*25 points*] Let $a$ and $b$ be real numbers with $a = \pm 1$. Verify that the map $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = ax + b$ is distance preserving: $|f(x) - f(y)| = |x - y|$ for all real numbers $x$ and $y$.

2. [*25 points*] Let $x, , y, z, w$ be noncoplanar points in 3-space (hence no three are collinear). Show that the lines $xy$ and $zw$ have no points in common but are not coplanar.

3. [*25 points*] Let $a, b, c, d, e$ be points in the plane such that $a * b * c$ and $a * d * e$. Using Pasch's Theorem for triangle $acd$, show that the open segments $(cd)$ and $(be)$ have a point in common. [*Hint:* Consider the line $eb$. Why does it not contain any points of $[ad]$? A rough sketch should provide some insight.]

4. [*25 points*] Suppose we are given isosceles triangle $ABC$ in the plane with $|AB| = |AC|$, and let $D$ be the midpoint of $[BC]$. Prove that the ray bisects $\angle BAC$: $|\angle BAD| = |\angle DAC| = \frac{1}{2}|\angle BAC|$. You may assume $D$ lies in the interior of $\angle BAC$ without proving this fact.

Extra page for use if needed