PREFACE

In this preface we shall give references for course materials and an extremely brief overview of the material to be covered in this course.

Course materials

The primary sources for the course will be the online notes, homework assignments and solutions to the latter which are, or will be, in the course directory:

http://math.ucr.edu/~res/math133

Specifically, full set of lecture notes and exercises will be posted in this directory, with solutions to the exercises, supplementary material and (if needed) still further commentary on the material to be posted as the course progresses.

The following previous texts for the course, which will be referred to as "Ryan" and "Greenberg" in the course notes and related materials, may be viewed as optional secondary references:

> P. Ryan, ***Euclidean and non – Euclidean geometry**: **An analytical approach.*** *Cambridge University Press*, Cambridge, U. K., and New York, NY, 1986. ISBN: 0–521–27635–7.
>
> M. J. Greenberg, ***Euclidean and non – Euclidean geometries: Development and history*** (Fourth Ed.). *W. H. Freeman*, New York, NY, 2007. ISBN: 0–716–79948–0.

For many sections of the course notes, there are suggested readings from portions of these books. The most important points of these passages will be summarized in suitably designated files to be uploaded into the course directory.

A slightly different (but related) perspective on basic Euclidean geometry is presented in the second half of the following text:

> Z. Usiskin, A. Peressini, E. A. Marchisotto, and D. Stanley, ***Mathematics for High School Teachers: An Advanced Perspective.*** *Prentice – Hall,* Upper Saddle River, NJ, 2002. ISBN: 0–130–44941–5.

The formal prerequisites for this course are the standard material in lower division mathematics courses, including calculus up to and including partial differentiation and (***this is particularly important!***) linear algebra through the theory of determinants (some but not all the options also include intermediate level material on dot or inner products, but the latter will be reviewed). Some background knowledge of set theory at the high school level or precalculus level is assumed; one source for background material on this subject is the text for the lower division course on discrete mathematics:

> K. H. Rosen, ***Discrete Mathematics and Its Applications*** (Fifth Ed.). McGraw – Hill, New York, 2003. ISBN: 0– 07293033– 0. *Companion Web site*: **http://www.mhhe.com/math/advmath/rosen/**

Also, ***reading and writing mathematical proofs will be an important part of the course.*** The following document contains the necessary background information on this topic:

http://math.ucr.edu/~res/math133/mathproofs.pdf

Reading this document before starting the material in the course notes or trying to work the exercises is strongly recommended.

Overview of the course

The basic objectives of this course are to cover the material in geometry specified in the State of California standards for mathematics teaching certification (see the first reference listed near the end of this Preface under the heading, ***State of California sites on standards for mathematics teaching***), to explain how other standard material in $\mathbf{K-12}$ mathematics classes fits into the setting developed here, and to cover the material described in the official catalog description of this course; the latter is given at the following online site:

http://catalog.ucr.edu/catalog2008/Mathematics.pdf

The notes for the course are split into five units. Linear algebra is a specific prerequisite for this course, and Unit $\mathbf{I}$ discusses the framework for studying geometrical problems by means of linear algebra. Some of this is covered in linear algebra, and often other basic mathematics courses discuss topics like "*proofs of geometric theorems using vectors*," so the main goal of this unit is to consolidate such ideas and take them a bit further. Perhaps the main points not seen in earlier courses are the interpretation of lines and planes as translates of vector subspaces and the notion of ***barycentric coordinates***, which turns out to be fundamentally important for both theoretical and computational purposes and is used often in later units of the notes. In Unit $\mathbf{II}$ we strengthen the connections between linear algebra and classical Euclidean geometry, explaining how concepts in one relate to concepts in the other and taking an twofold approach based upon a modern, logically complete version of the axioms for classical geometry and their interpretation in terms of linear algebra. Unit $\mathbf{III}$ is devoted to the standard material in classical geometry; we use both the classical or synthetic approach and the formulation of the analytic approach in terms of linear algebra, and the choices of approach at various points were made to make the discussions as clear, simple and straightforward as possible so that everything can be covered in a relatively coherent manner within the time limits of the course. In Unit $\mathbf{IV}$ we consider another type of geometry known as ***projective geometry***, which basically arose out of the theory of perspective drawing which artists developed during the late Middle Ages and early Renaissance. This work led to new types of geometric questions and a different approach to many basic ideas in geometry. Finally, Unit $\mathbf{V}$ of these notes will discuss the discovery and development of ***Non – Euclidean geometry***, beginning near the end of the 18^{th} century and continuing through virtually the entire 19^{th} century. Aside from the purely mathematical results coming from this work, the discovery of such new geometries had an enormous impact on the way that mathematicians thought about geometry in particular and more generally about the foundations of mathematics itself. Virtually all the main results and issues will be discussed in the final unit. There are separate file of exercises for each unit in the course directory.

There is probably too much material to cover in ten weeks, so Unit **IV** will ***not*** be part of the course. Fortunately, nothing from that unit is needed in Unit **V.**

The exercises files for this course contain a wide range of problems ranging from fairly routine to (at least) somewhat challenging. As in other mathematics courses, it is important to work or attempt as many of these as possible in order to reinforce one's grasp of the concepts and results in the notes. The following quote from the eminent physicist Freeman Dyson (1923 –), from his nontechnical book ***Disturbing the Universe***, expresses the point in emphatic terms:

> The difference between a text without problems and a text with problems is like the difference between learning to read a language and learning to speak it. I intended to speak the language [of physics] ... and so I worked my way through the problems.

Another way of expressing the same idea is to say that working the exercises is often indispensable for transforming a ***passive*** understanding of a subject into an ***active*** one.

Some further references

We have already included a few references to other books and online sites for further information, and we shall conclude this preface by listing a few sites dealing with State of California requirements, historical background, and online encyclopedia articles that are generally highly reliable; despite the justifiable controversy surrounding the reliability of some online ***Wikipedia*** artcles, the entries for standard, well – established topics in the sciences are generally very reliable, and I have checked out all the specific references in the course notes and found them to be accurate. Further comments about online references appear in the online file

http://math.ucr.edu/~res/math133/aabInternetresources.pdf

which is in the course directory. The web sites given in the notes often contain numerous links to other sites, and looking up the sales information for specific books on either **www.amazon.com** or **www.bn.com** will usually provide names of similar or related books. The approach, emphasis and coverage of undergraduate geometry texts are extremely variable, and in fact far more so than for books on other parts of mathematics at the undergraduate level; in particular, the difference in perspective between the books by Ryan and Greenberg is very substantial. In some cases the different perspectives of other books may be enlightening, but in others the differences may only be confusing, and unfortunately it is difficult to give many general recommendations.

STATE OF CALIFORNIA SITES ON STANDARDS FOR MATHEMATICS TEACHING

(*K* – 12 *and Teacher Certification*)

http://www.ctc.ca.gov/educator-prep/standards/SSMP-Handbook-Math.pdf

http://www.cde.ca.gov/ci/ma/cf/

http://www.cde.ca.gov/be/st/ss/documents/mathstandard.pdf

HISTORY OF MATHEMATICS SITES

http://www-groups.dcs.st-and.ac.uk/~history/index.html

http://aleph0.clarku.edu/~djoyce/mathhist/mathhist.html

http://dir.yahoo.com/science/mathematics/history/

http://en.wikipedia.org/wiki/History_of_mathematics

http://www.dean.usma.edu/math/people/rickey/hm/mini/textbooks.html

MATHEMATICS ENCYCLOPEDIA SITES

http://mathworld.wolfram.com

http://en.wikipedia.org/wiki/Main_Page

http://planetmath.org