

**Mathematics 133**

**Geometry**

**Course Notes**

**Fall 2013 Version**

**Department of Mathematics**

**University of California, Riverside**

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*Note:* “\*” means that there is no corresponding section in the notes, but there is such a section in the exercises for the course.

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`geometrynotes00.f13.pdf`, pp. *i-iv*, contains the Preface.

`geometrynotes01.f13.pdf`, pp. 1–30, contains Sections I.1–I.5.

`geometrynotes02a.f13.pdf`, pp. 31–58, contains Sections I.1–II.2.

`geometrynotes02b.f13.pdf`, pp. 59–86, contains Sections II.3–I.5.

`geometrynotes03a.f13.pdf`, pp. 87–120, contains Sections III.1–III.3.

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`geometrynotes04a.f13.pdf`, pp. 166–200, contains Sections IV.1–IV.3.

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