Math 133
Fall 2013

# STILL MORE EXERCISES FOR SECTION II.1

The following exercises deal with consequences of the incidence axioms. Assume that $(\mathbb{S}, \mathcal{P}, \mathcal{L})$ is a system which satisfies these axioms.

**J1.** Show that the axioms do not imply that every line or plane must have infinitely many points. [*Hint:* Take $\mathbb{S}$ to be a set with four points, let $\mathcal{P}$ and $\mathcal{L}$ be all subsets with two and three points respectively, and verify that this system satisfies all the axioms even though points and lines are finite sets.]

**J2.** Prove that if $L$ and $M$ are distinct coplanar lines, then there is a unique plane containing them. [*Hint:* The crucial point is that there is only one such plane.]

**J3.** Suppose that the three distinct points $A, B, C$ lie on the planes $P$ and $Q$, where $P \neq Q$. Prove that the set $\{A, B, C\}$ is collinear.

**J4.** If $P_1$, $P_2$ and $P_3$ are distinct planes, show that their intersection is either the empty set, a single point or a line. Give examples in $\mathbb{R}^3$ for which the intersections are of each (mutually exclusive) type.