NAME: ______________________________

Mathematics 133, Fall 2020, Examination 1

INSTRUCTIONS: Work all questions, and unless indicated otherwise give reasons for your answers. The point values for individual problems are indicated in brackets. Please submit a copy of your exam by electronic mail, one to **each** of the following addresses, by 11:59 P.M. on Thursday, November 12, 2020:

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Each problem should be started on a separate sheet of paper. Make sure your submission is readable; some smartphone photos might not have enough contrast. You may look at outside references such as course directory documents before starting this examination, and you may consult with other students, the teaching assistant or me about material related to this examination, but this assignment is **NOT** collaborative. The answers you submit must be your own work and nobody else's.

The top score for setting the curve will be 100 points.

#	SCORE
1	
2	
3	
4	
TOTAL	

1. [*25 points*] Assume that we are given a line L in the plane P, and also assume that A, B, C, D, E are five distinct points such that no three are collinear. Prove that at least two of these points lie on the same side of L in P. [*Note:* One or more points might lie on L.]

2. [*25 points*] (a) Let L and M be two lines in the coordinate plane $\mathbb{R}^2$ which meet at a single point. Suppose that a third line N is parallel to L. Show that M and N have a point in common.

(b) Suppose we are given $\angle ABC$ in the coordinate plane $\mathbb{R}^2$, and let L be a line in $\mathbb{R}^2$. Prove that L is not contained in the interior of $\angle ABC$. [*Hint:* Try to use part (a).]

3. [*25 points*] Suppose that we are given two triangles $\triangle ABC$ and $\triangle DEF$ in $\mathbb{R}^2$ such that $\triangle ABC \cong \triangle DEF$. Let $G \in (AC)$ and $H \in (DF)$ such that **either** $|\angle ABG| = |\angle DEH|$ **or** $|AG| = |DH|$. Prove that $\triangle GBC \cong \triangle HEF$. [*Hint:* Draw a picture.]

4. [*25 points*] The geometric reflection about the line joining $(0,0)$ and $(\cos\theta, \sin\theta)$ is the linear transformation from $\mathbb{R}^2$ to itself has matrix

$$S_\theta \quad = \quad \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$$

and the counterclockwise rotation by an angle of measure α is the linear transformation from $\mathbb{R}^2$ to itself with matrix

$$R_\alpha \quad = \quad \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} .$$

The composite of two reflections $S_\theta \circ S_\varphi$ is equal to a rotation matrix R_α. Express α in terms of θ and φ.

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