**NAME:** ______________________________

# Mathematics 133, Fall 2020, Examination 2

**INSTRUCTIONS:** Work all questions, and unless indicated otherwise give reasons for your answers. The point values for individual problems are indicated in brackets. Please submit a copy of your exam by electronic mail, one to **EACH of the following** addresses, by **11:59 A.M. on Wednesday, December 16, 2020:**

`rschultz@ucr.edu`   `amart136@ucr.edu`

Make sure that your email address is easily identifiable so that your exam does not get overlooked (addresses like `itsme@mysite.net` are problematic).

Each problem should be started on a separate sheet of paper. Make sure your submission is readable; some smartphone photos might not have enough contrast. For the numerical problem, you may use a calculator or simple programmable device to work the first part. You may also look at outside references such as course directory documents before starting this examination, and you may consult with other students, the teaching assistant or me about material related to this examination, but this assignment is **NOT** collaborative. The answers you submit must be your own work and nobody else's.

The nominal top score for setting the curve will be 150 points.

**1.** [*25 points*] Assume we are working in the coordinate plane. Let $\angle ABC$ be given, let $X$ be a point in the interior of $\angle ABC$, and let $Y \in (BA$. Assume also that the line $XY$ meets $(BC$ at a point $Z$. Which of the three points $X$, $Y$, $Z$ is between the other two? Give reasons for your answer.

**2.** [*25 points*] Suppose that we are working in a Euclidean plane $\mathbb{P}$, and let $ABCD$ denote a (convex) trapezoid with $AB||CD$. Assume further that $|AD| = |DC|$. Prove that $[AC$ bisects $\angle DAB$.

**3.** [*25 points*] Assume that we are given two angles $\angle DAB$ and $\angle DCB$ in a Euclidean plane $\mathbb{P}$, and suppose that we have $X \in (AB) \cap (CD)$. Prove the equation $|AX| \cdot |XB| = |CX| \cdot |XD|$.

**4.** [*25 points*] As in Quiz 2, take the last four digits $ABCD$ of your student identification number, and once again consider the point in the coordinate plane given by $X = (A+B, C+D)$; let $Y = (0,0)$ and $Z = (25,0)$. Find the orthocenter of $\triangle XYZ$. The proof of the theorem on orthocenters yields one way of solving this problem.

**5.** [*25 points*] Let $A$ be the set of all points in the coordinate plane $\mathbb{R}^2$ which are on either the nonnegative $x$-axis or the nonnegative $y$-axis (hence $A$ = all points of the form $(u, v)$ where either $u \geq 0$ and $v = 0$ or else $u = 0$ and $v \geq 0$). Describe the set $L$ of all points $(p, q)$ such that the (shortest) distance from $(p, q)$ to $A$ is equal to 1. Describe the points of $L$ in numerical terms (equations and inequalities involving the coordinates $p$ and $q$). There are four cases corresponding to the four quadrants of the coordinate plane.

**6.** [*25 points*] Assume that all points arising in this discussion lie in a hyperbolic plane $\mathbb{P}$. Suppose that we are given $\triangle ADE$ with $B \in (AD)$ and $C \in (AE)$ such that $|\angle ABC| = |\angle ADE|$. Is $|\angle ACB|$ greater than, equal to or less than $\angle AED|$? Prove that your answer is correct.