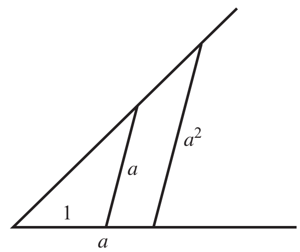
similar triangle, in which the side corresponding to 1 was a, the other side would be a^2.

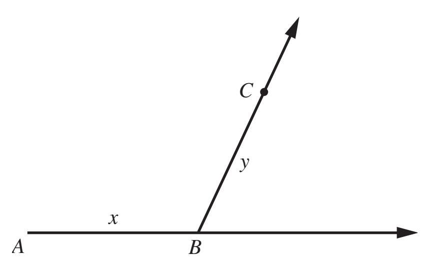


The problems that formed the central theme in *La Géométrie* were generalizations of the three- and four-line locus problems Pappus had propounded in his commentary on the *Conics* of Apollonius. In their original form the Pappus problems run: given four lines in a plane, find the locus of a point that moves so that the product of the distances from two fixed lines (along specified directions) is proportional to the square of the distance from the third line [three-line locus problem], or proportional to the product of the distances from the other two lines [four-line locus problem]. Whereas Pappus had stated without proof that the required locus was one of the conic sections, Descartes showed this algebraically. Subsequently, Newton solved the problem geometrically in his *Principia* (1687).

Descartes began his attack on the problem by choosing one of the given lines, say, AB, and a fixed point on it, say, A (he selected what would later be called an axis of coordinates and an origin). From an arbitrary point C of the locus sought, a straight line CB was drawn to AB, meeting it at a given angle. The lines AB and BC were then the quantities that would determine the position of C, and he called them x and y:

> I would simplify matters by considering one of the given lines and one of those to be drawn (for example, AB and BC) as the principal lines to which I shall try to refer all others. Call the segment of the line AB between A and B, x and call BC, y.

The lengths of the other lines were expressed in terms of x and y; and by the conditions of the problem. Descartes combined these, to arrive at an equation of the curve upon which C would have to lie.



What we have here is the germinal idea of a coordinate system in which the position of a point in the plane is defined by its distances, x and y, from two fixed axes. Descartes was choosing what in current language is an oblique coordinate system, although he did not formally introduce a second axis, the y-axis. Nowhere in *La Géométrie* does the modern rectangular coordinate system appear. Descartes's presentation differed from that now current also in his use of only positive values of x and y, that is, by his restriction

Descartes was thus led to one proposition so sound that it could not be doubted, the certainty of his own existence; for doubt itself is an act of thought and thought does not take place without a thinker. He enunciated this in the most famous sentence in philosophy, one that has been the subject of numerous commentaries: “Je pense, donc je suis.” (I think, therefore I am.) Having satisfied himself of the existence of a thinking being, Descartes passed on to a search for other propositions that appeared equally self-evident and irrefutable. For him, there was no greater guarantee of the truth of a proposition than that it should survive the most careful scrutiny of his own independent criticism. “We ought never to allow ourselves to be persuaded of the truth of anything unless on the evidence of our reason,” Descartes wrote. This unbounded confidence in the capacity of human reason helped launch the Great Debate between faith and reason that was to preoccupy most Western Europeans in the century to come.

Inventing Cartesian Geometry

Three appendixes to the *Discours* were actual illustrations of Descartes’s new method of discovering scientific truths. Although the *Discours* was intended to be a preface to *La Dioptrique*, *Les Météores*, and *La Géométrie*, history has completely reversed the sequence; and today the *Discours* is studied by students of modern philosophy, while these works on science are virtually ignored. *La Dioptrique* (*Dioptrics*) deals with the nature and properties of light, including an account of the law of refraction, the anatomy of the human eye, and the shape of lenses best adapted for the newly invented telescope. *Les Météores* (Meteorology) aims at a scientific explanation of atmospheric phenomena; it is concerned with such topics as how snow crystals are formed, the size of raindrops, the cause of thunder and lightning, and the formation of the rainbow. Of the three essays accompanying the *Discours*, the third, *La Géométrie* (Geometry), is the one in which Descartes made his great and lasting contribution to pure mathematics. In the *Géométrie*, he combined the methods of algebra and geometry to produce the new field of analytic geometry. The English philosopher John Stuart Mill called this “the greatest single step ever made in the progress of the exact sciences.” Tradition holds that the idea of analytic geometry came to Descartes while he watched a fly crawl along the ceiling of his room near a corner; his immediate problem became expressing the path of the fly in terms of its distance from the adjacent walls. The story is more agreeable fable than fact.

Of the three parts of *La Géométrie*, the first two are devoted mainly to applying algebra to geometry, while the third treats the theory of equations. Book I bears the title *Problems Which Can Be Constructed by Means of Circles and Straight Lines Only*. At the threshold of the work, Descartes introduced the algebraic notation still in use today. The last letters of the alphabet, x, y, and z, designate unknown quantities, and the first letters of the alphabet stand for constants. Descartes was perhaps the first to use the same letter for both positive and negative quantities. Our modern exponential notation for powers is also found here. In a more radical step, Descartes broke with Greek tradition by divorcing numbers from reference to physical quantity. Instead of interpreting a^2 (or aa as he wrote it) and a^3, for example, as an area and a volume, he considered them nothing more than lines. For Descartes, a^2 was simply the fourth term in the proportion $1 : a = a : a^2$ and as such could be represented by a line once a was given. To devise a construction that corresponded to the proportion $1 : a = a : a^2$, he arbitrarily chose a unit length 1, to which all other lengths were referred. Then a triangle with sides 1 and a was drawn; in a

LA

GEOMETRIE.

LIVRE PREMIER.

Des probleſmes qu'on peut conſtruire ſans y employer que des cercles & des lignes droites.

Tous les Probleſmes de Geometrie ſe peuuent facilement reduire a tels termes, qu'il n'eſt beſoin par aprés que de connoiſtre la longeur de quelques lignes droites, pour les conſtruire.

Et comme toute l'Arithmetique n'eſt compoſée, que de quatre ou cinq operations, qui ſont l'Addition, la Souſtraction, la Multiplication, la Diuiſion, & l'Extraction des racines, qu'on peut prendre pour vne eſpece de Diuiſion : Ainſi n'a-t'on autre choſe a faire en Geometrie touchant les lignes qu'on cherche, pour les preparer a eſtre connuës, que leur en adiouſter d'autres, ou en oſter, Oubien en ayant vne, que ie nommeray l'vnité pour la rapporter d'autant mieux aux nombres, & qui peut ordinairement eſtre priſe a diſcretion, puis en ayant encore deux autres, en trouuer vne quatrieſme, qui ſoit à l'vne de ces deux, comme l'autre eſt a l'vnité, ce qui eſt le meſme que la Multiplication ; oubien en trouuer vne quatrieſme, qui ſoit a l'vne de ces deux, comme l'vnité

Commēt le calcul d'Arithmetique ſe rapporte aux operations de Geometrie.

First page of Descartes's *La Géométrie* (1637). *(Reprinted by permission of Open Court Publishing Company, La Salle, Illinois, from* The Geometry of René Descartes*, translated by David Eugene Smith and Marcia L. Latham.)*

fashion from the most universal principles, it would be possible to deduce everything rationally knowable.

The starting point for Descartes's thought was to discover the simplest ideas or principles, those of which there could be no doubt. Because he had lost all con dence in traditional teachings. Descartes began by breaking away from authority altogether in matters of science and philosophy, deliberately rejecting all entrenched dogmas and doctrines. In his own words from the *Discours*:

> I thought that I ought to reject as absolutely false all opinions in regard to which I could suppose the least ground of doubt, in order to ascertain whether after that there remained anything in my belief that was wholly indubitable.

cold. She proposed to Descartes that they meet three times a week, always at five o'clock in the morning, when her mind was unfatigued and she felt the most energetic. For two months Descartes conformed to his royal pupil's schedule, walking in the winter dawn from his rooms to the ice-cold library. His own lifelong routine was radically changed; as a foreign Catholic at a Lutheran court he felt isolated and homesick. "It seems to me," he wrote to his friend the Comte de Brégy, "that men's thoughts freeze here in winter just like water." The rigors of one of the bitterest winters in memory proved too much for Descartes's constitution, which had never been robust. On February 1, 1650, he caught a cold that rapidly developed into pneumonia, and he died after 10 days of suffering and delirium. He was buried where Catholics were usually interred, in a cemetery set aside for infants who died before baptism. Fifteen years later his remains (except for the right hand, which was kept as a memento by the official who arranged the transaction) were conveyed back to France, where a magnificent monument was erected to his memory in the Church of Saint Genevieve. As Descartes's doctrines were by then under the ban of both the Church and the universities, the funeral oration was prohibited by a court order, which arrived during the funeral service.

The year 1637 saw the publication of the work that is considered the most significant of Descartes's writings: *Discours de la Méthode pour bien conduire sa Raison et chercher la Vérité dans les Sciences* (Discourse on the Method of Rightly Conducting the Reason in the Search for Truth in the Sciences), with its scientific appendages *La Dioptrique*, *Les Météores*, and *La Géométrie*. The *Discours* is not, as commonly supposed, a formal philosophical treatise but a short autobiographical résumé of Descartes's progress in arriving at his method. Its first edition had 78 pages, roughly a sixth of the entire work. It was written in his native tongue, though traditionally Latin was used for learned subjects, and it showed at once the power and precision of the vernacular as a vehicle for expressing highly complicated philosophical and scientific thoughts. (In *Principia Philosophiae*, Descartes reverted to Latin to make the work more acceptable to the Church and the universities.) Descartes's use of the French language speeded the diffusion of his ideas. The work was widely read; but though the *Discours* brought fame to its author, the fortune went to the book's printer. The printer paid a small price indeed for one of the landmarks in Western thought. Descartes had asked only to be given, instead of royalties, 200 free copies of the new book for distribution to his friends.

The whole of Descartes's philosophy of "systematic doubt" as expounded in the *Discours* is dominated by his pursuit of certainty. The certainty of mathematics, he delighted to repeat, consists of this—it starts with the simplest elements whose truth is recognized, and then proceeds by the process of deduction from one evident proposition to another. Mathematics should therefore be a model for other branches of study. To let Descartes speak for himself:

> The long chains of simple and easy reasonings by means of which geometers are accustomed to reach the conclusions of their most difficult demonstrations led me to imagine that all things, to the knowledge of which man is competent, are mutually connected in the same way, and that there is nothing so far removed from us as to be beyond our reach, or so hidden that we cannot discover it, provided only we abstain from accepting the false for the true, and always preserve in our thoughts the order necessary for the deduction of one truth from another.

The character of the reasoning of mathematics rather than the results was what so impressed Descartes. And he was anxious to see whether, by arguing in a mathematical marvelous science whose foundations he found on this memorable day. Some authorities are inclined to believe that he formulated the principles of analytic geometry; others feel that Descartes conceived a complete reform of philosophy based on the methods of mathematics. As Bertrand Russell observed, “Socrates used to meditate all day in the snow, but Descartes’s mind only worked when he was warm.”

By 1628, having grown weary from years of aimless wandering through Holland, Germany, Hungary, and Italy, Descartes settled down to what might be called the productive period of his life. Holland, which had recently won independence after a protracted struggle with Spain, seemed the country best fitted to offer the tolerance and tranquility Descartes needed to pursue his researches. There, in great seclusion (barring three brief visits to France to look after family affairs), he meditated and wrote for 20 years. Until then he had published nothing. Descartes conceived therefore of writing an almost encyclopedic treatise on physics, which he chose to call *Le Monde* (The World). The time from 1629 to 1633 was occupied with building up a cosmological theory of vortices to explain all natural phenomena. On the eve of the completion of *Le Monde*, he learned that Galileo’s *Dialogue on the Two Chief Systems of the World*, published the previous year, had earned the censure of the Church. It was clear that the earth was not to be summarily dismissed from its position as the immovable center of the solar system. His own work, affirming as it did the heliocentric hypothesis, would have made him equally guilty with Galileo, so Descartes prudently abandoned the project. The publication of *Le Monde* had to wait until 1664, well after his death.

It was not moral weakness that forced Descartes to suspend publication of *Le Monde*, but rather that he never ceased to regard himself as a sincere and devoted Roman Catholic. He wrote sadly to Mersenne, “This has so strongly affected me that I have almost resolved to burn all my manuscript, or at least show it to no one. But on no account will I publish anything that contains a word that might displease the Church.” Not that the fruits of his labor were withheld from the world, for Descartes did not destroy his papers as he first threatened to do. The ideas contained therein, modified but not abandoned, had their presentation to the public in his first principal published work, the *Discours de la Méthode* (1637). Although the *Discours* included a summary of *Le Monde*, Descartes so sidestepped the controversy over Copernicanism that one could glean little from it concerning his cosmology; in particular, any mention of vortices was studiously avoided. Finally, in 1644, the *Principia Philosophiae* was issued, in which he explained at some length the formation of the physical world, by “gradual and natural means” out of matter and motion. Descartes’s new “mechanical philosophy” quickly became the rage, a dominant feature of discussion in intellectual circles.

By 1649, Descartes’s reputation had been established throughout Europe, and he was invited by Queen Christina of Sweden, the daughter of Gustavus Adolphus, to visit her court to tutor her in philosophy. She also suggested that he might help her in planning an academy of sciences that would rival the best in Europe. When Descartes had misgivings about living in “the land of bears amongst rocks and ice,” the young queen (she was then but 22 years old) dispatched an admiral to coax him and then a Swedish warship to fetch him. Accepting the invitation was a fatal decision on Descartes’s part. It is even said that a presentiment of death came over him as he prepared for the journey.

Descartes was received with every honor and had no cause for complaint until the time drew near for his personal instruction of the queen to begin. From childhood on, Christina had slept no more than five hours a night, and she was indifferent to heat or

René Descartes
(1596–1650)

(Smithsonian Institution.)

this habit; throughout the rest of his life (except for one unfortunate incident that may well have hastened his death), Descartes preferred to rise late, spending the early hours in bed meditating and writing. Indeed, when he visited Pascal in 1647, Descartes stated that the only way he could do good work in mathematics and preserve his health was never to allow anyone to get him up in the morning before he felt so inclined.

On leaving school in 1612, Descartes followed the usual path of a young man of wealth living in France by going to Paris to taste the pleasures of its social life. This phase did not last long, for in Paris, he renewed his schoolboy friendship with that most indefatigable of learned gossips, the good Father Mersenne. Mersenne soon rekindled Descartes's interest in serious study, and in almost cloistral retirement they devoted two years to mathematical investigation. Although the younger Descartes had no deep inclination to follow his father's profession, he then entered the University of Poitiers, where he earned a degree in law in 1616.

In 1617 Descartes, then 21 years old and tired of textbooks, decided to learn more about the world at firsthand. He enlisted in the army as a gentleman volunteer, first joining the troops of Prince Maurice of Nassau in Holland and afterward taking service under the Duke of Bavaria. There is no evidence of any real soldiering on Descartes's part, only years of leisure, in which he had time to pursue his favorite studies. The night of November 10, 1619, while in winter quarters with the Bavarian army along the Danube, was critical in Descartes's life. He escaped the cold by shutting himself up alone all day in a "*poêle*"—literally a stove, actually an overheated room. Tired from the heat, he dreamed three feverish dreams, in which he discovered "the foundations of a marvelous science." At the same time his future career as a mathematician and philosopher was revealed to him. (Near the close of the final dream, as Descartes tells us, he saw a book opened at a passage of the Latin poet Ausonius, containing the words "Which way shall I follow?" As the dream continued, an unknown man handed him a bit of verse beginning, "Is and is not," which he understood as representing truth and falsehood in human knowledge.) Descartes neglected to specify the exact nature of the

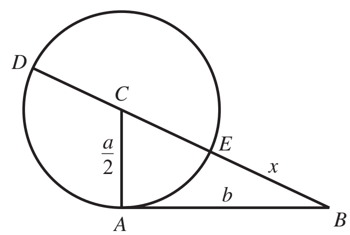
(b) $$2/\pi = 2/\pi \cdot \lim_{n\to\infty}\left(\frac{\pi/2^n}{\sin \pi/2^n}\right) = \lim_{n\to\infty}\left(\frac{1}{2^{n-1}\sin\pi/2^n}\right).$$

(c) $$2/\pi = \cos\frac{\pi}{4}\cos\frac{\pi}{8}\cos\frac{\pi}{16}\cdots = \left(\frac{\sqrt{2+2\cos\pi/2}}{2}\right)\left(\frac{\sqrt{2+2\cos\pi/4}}{2}\right)\left(\frac{\sqrt{2+2\cos\pi/16}}{2}\right)\cdots = \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2+\sqrt{2}}}{2}\cdot\frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2}\cdots.$$

12. Vièta solved the quadratic equation $x^2 + ax = b$ by substituting $x = y - a/2$. This produces a quadratic in y in which the rst-de gree term is missing. Use Vièta's method to solve the quadratic equations:

(a) $x^2 + 8x = 9$.
(b) $x^2 + 10x = 144$.
(c) $x^2 + 12x = 64$.
(d) $3x^2 + 10x = 32$. [*Hint:* Multiply both sides by 3 and let $z = 3x$.]

13. To solve the equation $x^2 + ax = b^2$ geometrically, René Descartes would have used the method as described. Draw a line segment AB of length b and at A erect a perpendicular AC of length $a/2$. With C as center, construct a circle of radius $a/2$ and draw a line through B and C, intersecting the circle at points D and E. Prove that the length of the segment BE is the value of x that satis e s $x^2 + ax = b^2$.



8.2 Descartes: The *Discours de la Méthode*

The Writings of Descartes

Among the principal movers in the seventeenth-century scienti c revolution, René Descartes must certainly be included.Through the publication of *La Géométrie*, which made analytic geometry known to his contemporaries, Descartes is generally acknowledged to have laid the foundations for the growth of mathematics in modern times. This rst really great advance beyond the techniques known to the ancients changed the face of mathematics and led, within a generation, to the development of the calculus by Newton and Leibniz. It is not too much to say that Descartes's career marks the turning point between medieval and modern mathematics.

René Descartes (1596–1650) was born at La Haye, a small town about 200 miles southwest of Paris, in the province of Touraine. His father belonged to the lesser nobility. He was a councilor at the Parlement of Brittany—in effect, a provincial judge. Descartes went through the normal upbringing of a gentleman of that time. At age eight he was placed in the lately founded Jesuit College of La Flèche, perhaps the most illustrious school in which a student could enroll. There he came to know Marin Mersenne, who was seven or eight years older. The rst ve years of the curriculum at La Flèche were devoted to the traditional course in languages and the humanities. The nal three years embraced logic, philosophy, physics, and mathematics. Mathematics, because of the certainty of its demonstrations, was the only subject that really satis ed Descartes, even at this early age.

Descartes's health was delicate during infancy and childhood, and he was not expected to live long. His teachers at La Flèche, recognizing this physical weakness, treated him with exceptional consideration; regular attendance at lectures was not required of him, and he was allowed to lie in bed each morning as late as he pleased. He never lost

8.1 Problems

1. Solve the following problems from the *Treviso Arithmetic*:

(a) A man nds a purse with an unknown number of ducats in it. After he spends 1/4, 1/5, and 1/6 of the amount, 9 ducats remain. It is required to nd out how much money was in the purse.

(b) A hare is 150 paces ahead of a hound which pursues him. The hare covers 6 paces each time the hound covers 10. It is required to know how many paces the hound has made when he overtakes the hare.

(c) The Holy Father sent a courier from Rome to Venice, commanding him that he should reach Venice in 7 days. The most illustrious Signoria of Venice also sent another courier to Rome, who should reach Rome in 9 days. And from Rome to Venice is 250 miles. It happened that by the order of these lords the couriers started their journeys at the same time. It is required to nd in how many days they will meet.

2. From Nicolas Chuquet's *Tripary*, 1484: I am owed 3240 orins by a debtor who pays me 1 orin the rst day, 2 the second day, 3 the third day, and so on. In how many days will the debt be paid off?

3. From Robert Recorde's *The Whetstone of Witte*, 1557: A captain marshalls his army in a square formation. When the square is of one size, he has 284 men too many. But when he rearranges them in a square one man more on a side than before, he lacks 25 men. How many men does he have?

4. From Christoph Clavius's *Algebra*, 1608: If I gave 7 lire to each beggar that came to my door, I would have 24 lire left. But if I tried to give them 9 lire apiece, I would be lacking 32 lire. How many beggars came to my door, and how many lire did I have?

5. The two problems below are found in the *Rechnung* (1552) of Adam Riese. Solve them.

(a) A son asks his father how old he is. The father answers him by saying: If you were already as much, half again as much, and a fourth again as much older than you are now, and one more year, you would be 100 years old. The question is, how old is the son?

(b) Seven orins from Padua may be exchanged for 5 at Venice, and 10 orins at Venice are worth 6 at Nuremberg. Also 100 orins from Nuremberg are worth 73 at Köln. What is the value in Köln of 100 Paduan orins?

6. Use Napier's rods to multiply 458 by 79.

7. From the de nition of Napier's logarithm, derive the formulas:

(a) $\text{Nap.log}(M/N) = \text{Nap.}\log M - \text{Nap.}\log N + \text{Nap.}\log 1.$

(b) $\text{Nap.}\log M^r = r\ \text{Nap.}\log M + (1-r)\text{Nap.}\log 1.$

8. If $N = 10^7(1-10^{-7})^n$, justify the equations below relating the Naperian logarithm of N to its natural logarithm:

$$\begin{aligned}\text{Nap.}\log N &= \log_{1-10^{-7}}(10^{-7}N)\\ &= 10^7 \log_{(1-10^{-7})^{10^7}}(10^{-7}N)\\ &= 10^7 \log_{1/e}(10^{-7}N) = 10^7 \log_e(10^7/N).\end{aligned}$$

9. Find an approximation to the number N such that $\text{Nap.log } N = 6$.

10. The transformation

$$\log x = \frac{\text{Nap.}\log 1 - \text{Nap.}\log x}{\text{Nap.}\log 1 - \text{Nap.}\log 10}$$

converts Napier's logarithm to Briggs's common logarithm. Show that:

(a) $\log 1 = 0$, $\log 10 = 1$.

(b) $\log xy = \log x + \log y$, $\log(x/y) = \log x - \log y$.

(c) $\log x^r = r \log x$.

(d) $\log 10^n x = n + \log x$.

11. François Vièta's trigonometric skill led to his discovery (1593) of an in nite-product expansion for π in terms of square roots. Supply the missing details in his derivation:

(a)
$$\begin{aligned}1 &= \sin \pi/2\\ &= 2\sin\frac{\pi}{4}\cos\frac{\pi}{4} = 2^2\left(\sin\frac{\pi}{8}\cos\frac{\pi}{8}\right)\cos\frac{\pi}{4}\\ &= 2^3\left(\sin\frac{\pi}{16}\cos\frac{\pi}{16}\right)\cos\frac{\pi}{4}\cos\frac{\pi}{8}\\ &= 2^{n-1}\sin\frac{\pi}{2^n}\cos\frac{\pi}{4}\cos\frac{\pi}{8}\cos\frac{\pi}{16}\cdots\cos\frac{\pi}{2^n}.\end{aligned}$$

The riddle of the orbit of Mars engrossed Kepler's attention for the next eight years. Tycho's very accurate measures of the position of Mars relative to the sun enabled Kepler to test various hypotheses and cast them aside when they proved incompatible with the observed movement. With a Pythagorean craving for simplicity, he felt sure that the orbit was a circle. It was only after many failures to t the data to a circular orbit that he began to suspect that it must be some other closed path. For a long time, Kepler was inclined to believe that it was an oval, shaped like an egg. He tried various sorts of ovals, but none eliminated the discrepancies between his tentative theories and Tycho's observations. Years of work and disappointment nally forced him to the conclusion that only an elliptical orbit, with the sun occupying one of the two foci, satis ed Tycho's data. The same was presumably true for all other planets, because the harmony of nature demanded that all "have similar habits." This was Kepler's celebrated rst law. Another conclusion he extracted from the astronomical data was that the speed with which a planet traversed its elliptical orbit varied in a regular pattern, accelerating with approach to the sun and decelerating with departure from the sun. From this he was led to another pillar of celestial mechanics, Kepler's second law: The line drawn from the sun to a planet sweeps over equal areas in equal times.

The full history of his investigation of Mars, together with the laws just stated, was published in 1609 in a long book called *Astronomia Nova*. After 10 years' further effort, Kepler arrived at a relation, his third and last great law of planetary motion, connecting the times of revolution of any two planets with their respective distances from the sun. The ground was thus prepared for the later achievements of Isaac Newton, who was able to prove mathematically not only that the behavior implicitly extended by Kepler to all the planets agreed with observation but also that no other behavior was possible.

Kepler's celebrated results can be described thus:

1. The planets move in elliptical orbits with the sun at one focus.

2. Each planet moves around its orbit, not uniformly, but in such a way that a straight line drawn from the sun to the planet sweeps out equal areas in equal time intervals.

3. The squares of the times required for any two planets to make complete orbits about the sun are proportional to the cubes of their mean distances from the sun.

Archimedes is reported to have boasted, "Give me a place to stand and I will move the world." The observations Tycho Brahe had gathered became for Kepler a place to stand, and he did move the world. His three laws, which established the rst correct principles of planetary mechanics, overturned medieval cosmology and much of Aristotelian physics. From ancient times through the days of Copernicus and Tycho the idea that a planet's orbit must be a circle had been unchallenged. Indeed, a circle was accepted as the perfect geometrical form, and perfection was accepted as the normal state of heavenly affairs. That the real orbits of planets were ellipses was a triumph for the new astronomy and the right of Western scientists to pursue their investigations independent of theological doctrines.

One fundamental question could not yet be answered: What held the planets in their courses? As we shall see, the vortex theory of Descartes and Newton's theory of universal gravitation were responses to this challenge.

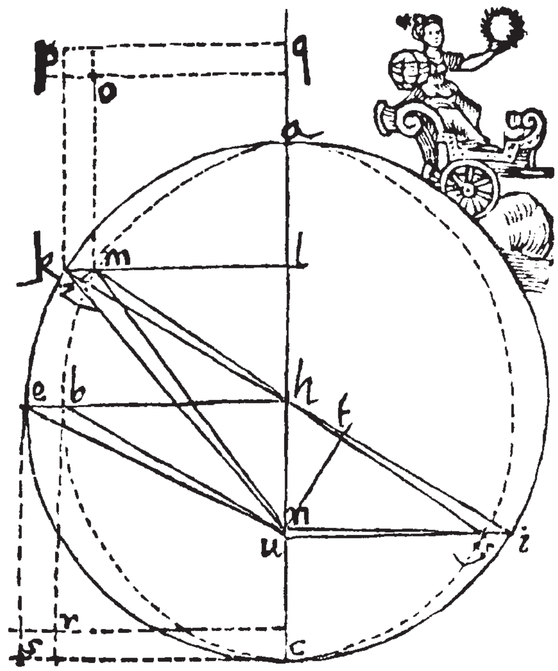
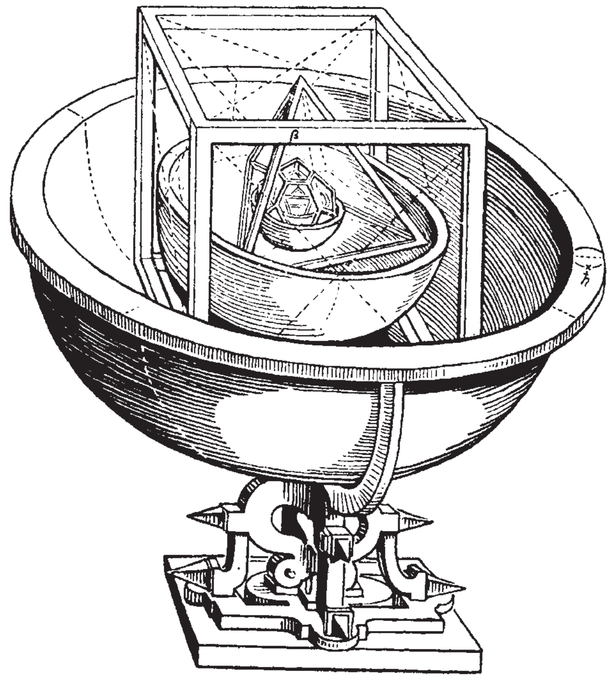


Diagram used by Kepler to demonstrate the elliptical orbit of Mars. From his *Astronomia Nova* (1609). *(From* A Short History of Astronomy *by Arthur Berry, 1961, Dover Publications, Inc., N.Y.)*

at the table through the rest of the meal, not wishing to leave before the other guests did. He died 11 days later, after intense suffering. On his deathbed, he turned to Kepler in particular and begged him to complete some of his tables on planetary motion as quickly as possible. It is also said that, in the delirium that preceded his death, Tycho repeated several times, "I hope that I will not appear to have died in vain." Kepler did not gain possession of Tycho's instruments, and they were inadvertently burned. Because Kepler's poor eyesight made him an indifferent observer, the loss was of little practical consequence. The most important scienti c inheritance Tycho left him was a vast wealth of astronomical observations of unparalleled accuracy. In the hands of Kepler, this store of information was to produce the next great advance in mathematical astronomy.

Shortly before Tycho's death, Kepler received the title of Imperial Mathematician from the Emperor Rudolf; and now he succeeded to Tycho's position as of cial astronomer to the emperor. But his royal master had the habit of paying his salary only rarely or in part, so that Kepler was forced to earn additional income by casting the horoscopes of eminent men. "Mother Astronomy would certainly starve if the daughter Astrology did not earn their bread," he is reported to have said.



Kepler's model, showing the relations between the planetary spheres and regular geometric solids. From his *Mysterium Cosmographicum* (1596). *(Extract taken from* A History of Science, Technology and Philosophy in the 16th and 17th Centuries*, by A. Wolf. Reproduced by kind permission of Unwin Hyman Ltd.)*

Tycho Brahe (1546–1601) was the pioneer of accurate astronomical observation. With the help of the king of Denmark, he had constructed in 1576 a splendid observatory on a 2000-acre island near Copenhagen. He equipped this with the most accurate instruments possible, including a 37-foot quadrant for measuring altitudes. None of the instruments had lenses, for the telescope was not invented until around 1600. With immense patience and skill, he labored for some 20 years compiling a vastly more extensive and incomparably more precise set of records than any of his predecessors had possessed. When the king died, the patronage was not extended by his successor, so Tycho moved to Prague in 1599, taking his most portable instruments along. There he entered the service of the eccentric Rudolf II, Holy Roman Emperor and the greatest patron of astrologers and alchemists in Europe. Kepler accepted Tycho's invitation to join him and arrived in Prague early in the following year. It was a fortunate alliance. Tycho was a splendid observer but a poor mathematician, whereas Kepler was a splendid mathematician but a poor observer.

Toward the end of 1601, after drinking copiously at a dinner party, Tycho was suddenly felled by a burst bladder. Adhering to the strict etiquette of the day, he remained