Math 133
Fall 2020

More exercises for Sections III.3 – III.8

1. A convex quadrilateral $ABCD$ is said to be a *kite* if $|AB| = |AD|$ and $|CB| = |CD|$.

(a) Find examples of kites which are not parallelograms.

(b) Suppose that a convex quadrilateral is both a kite and a parallelogram. What stronger conclusion can be drawn? (For example, must it be a square or rectangle, or ...?)

(c) If the convex quadrilateral $ABCD$ is a kite, prove that the area of the associated solid kite (the kite $ABCD$ plus its interior) is equal to $|AC| \cdot |BD|$.

2. One way of modelling home plate on a baseball field is to construct a polygon with vertices $A = (0, 0)$, $B = (p, 0)$, $C = (p + q, q)$, $D = (q, p + q)$ and $E = (0, p)$, where $p, q > 0$. Prove that these five points, in the given order, form the vertices of a convex pentagon.

3. Suppose that we are given an acute angle $\angle ABC$, and let S be the set (or locus) of all points X in the interior of $\angle ABC$ such that the distance from X to BA is half the distance from X to BC. Prove that S is an open ray originating at B.

4. Assume we are working in a Euclidean plane, and let A, B, C and D are four points such that no three are collinear and both C and D lie on the same side as the line AB. Explain why we cannot have $\Delta CAB \sim \Delta DAB$ or $\delta CBA \sim \Delta DBA$. [*Hint:* Why are $[AC$ and $[AD$ distinct?]

5. Suppose that we are given real numbers satisfying $b \geq a > 0$. For which values of $c > 0$ is there a triangle whose sides have lengths equal to a, b, c? Give reasons for your answer.