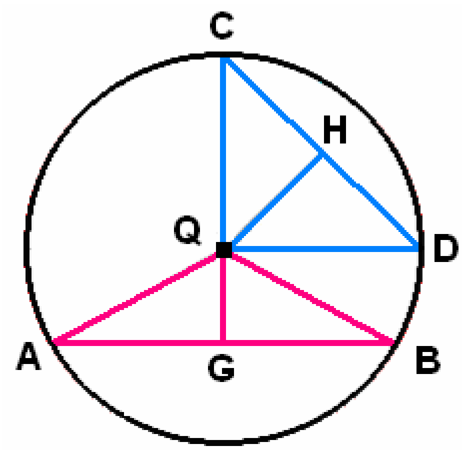
FIGURES FOR SOLUTIONS TO SELECTED EXERCISES

III : Basic Euclidean concepts and theorems

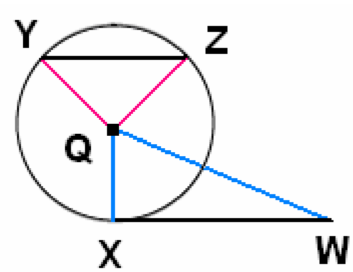
III.6 : Circles and classical constructions

III.6.1.



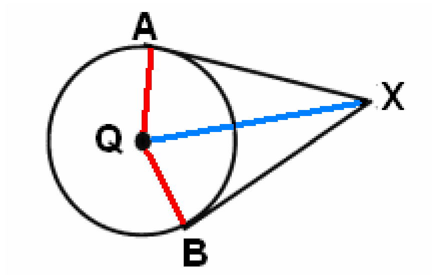
The lines **QG** and **QH** are assumed to be the perpendicular bisectors of **[AB]** and **[CD],** where all four endpoints lie on some circle whose center is the point **Q.** In this exercise the objective is to show that d(**Q, G**) = d(**Q, H**) if and only if d(**A, B**) = d(**C, D**), and d(**Q, G**) < d(**Q, H**) if and only if d(**A, B**) > d(**C, D**).

III.6.2.



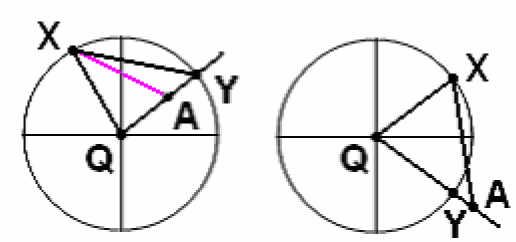
We are given a circle with center **Q** and two lines **XW** and **YZ** such that **XW** meets the circle at **X** and nowhere else, and the circle meets **YZ** in the two points **Y** and **Z.** The objectives are to show that **X** is the foot of the perpendicular from **Q** to **XW,** and that **YZ** is not perpendicular to either **QY** or **QZ** (and in fact, as suggested by the drawing, ∠**QYZ** and ∠**QZY** are acute). For the first part, recall that the foot **V** of the perpendicular is the closest point to **Q,** and explain why either **V** = **X** or else **V** lies inside the circle (so that the Line – Circle Theorem would apply to **XW** and the given circle).

III.6. 3.



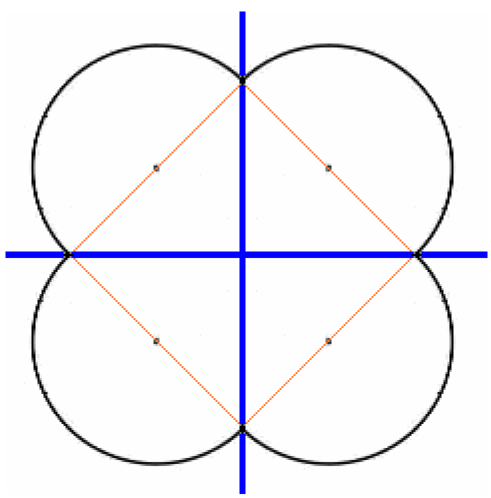
The tangent lines to the circle are perpendicular to the radii at the point of contact, and all segments joining the center of the circle to points on the circle have equal length. In this problem the objective is to prove that $d(\mathbf{X}, \mathbf{A}) = d(\mathbf{X}, \mathbf{B})$.

III.6.6.



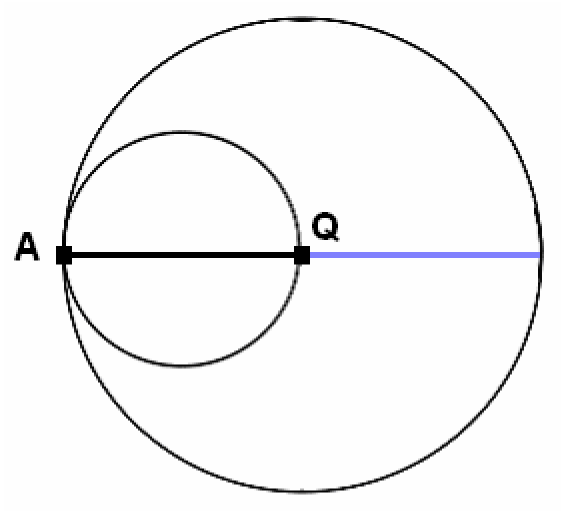
The objective is to prove that the distance from **A** to **X** is greater than the distance from **A** to **Y**, where **Y** is the unique point at which **[QA** meets the circle. There are two cases, depending upon whether the point **A** lies in the interior or exterior of the circle.

III.6.12.



The coordinate axes are drawn in blue, and the set **S** is the curve drawn in black; it is a union of four semicircles whose diameters are drawn lightly in orange.

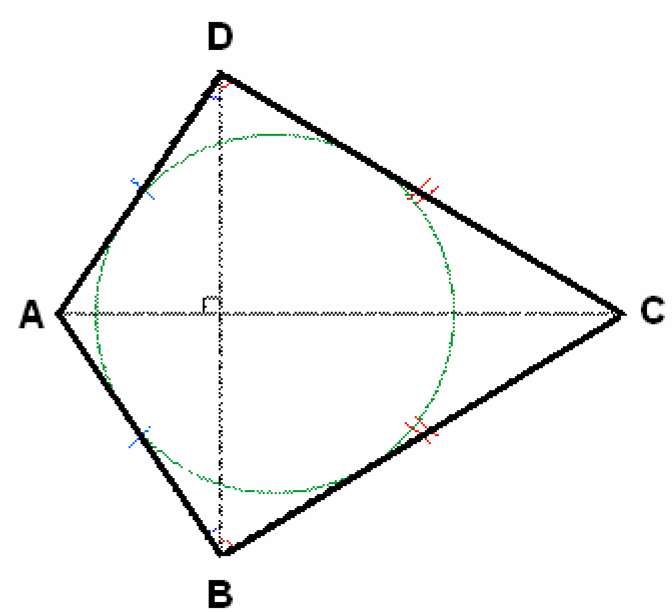
III.6.13.



The large circle is Γ, and the objective is to show that the set Ω is equal to the smaller circle for which **[AQ]** is a diameter.

III.6.15.

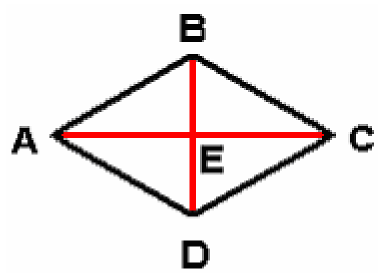
The drawing for Exercise **III.3.22** is also useful for this exercise. Here it is:



One key step in the argument is to prove that the bisectors for the angles at vertices **B** and **D** meet at a point on the line **AC.**

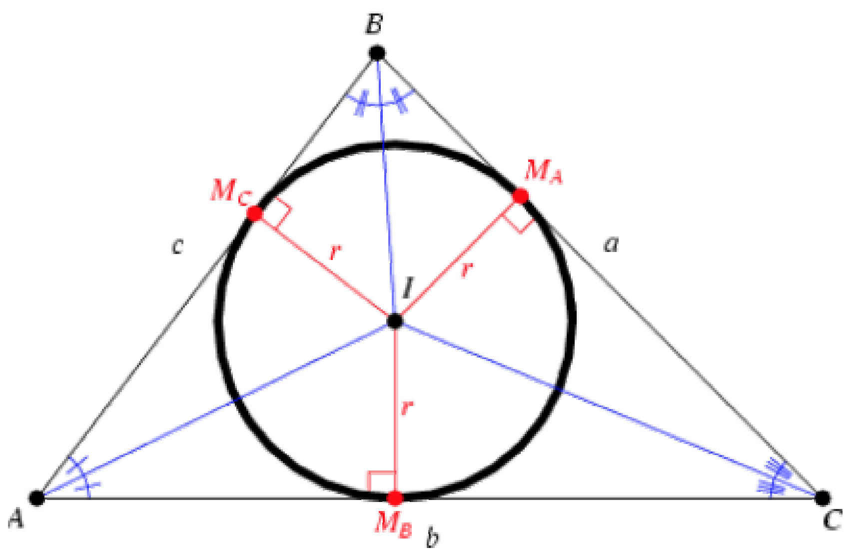
III.7 : Areas and volumes

III.7.1.



The points **A, B, C, D** form the vertices of a rhombus**,** so the diagonals are perpendicular and bisect each other**.** One way to compute the area of the closed region bounded by the rhombus is to split it into two triangles along **AC** and to use the observations of the preceding sentence in order to express the areas of the triangles in terms of the lengths of the diagonals of the rhombus**.**

III.7.4.



Here is the diagram for Theorem **III.4.8** that is mentioned in the hint**.** Note the decomposition of the region ◆**ABC** into six solid triangular regions with one vertex in common**,** and the altitude of each triangle from this common vertex is equal to the inradius ***r*****.**