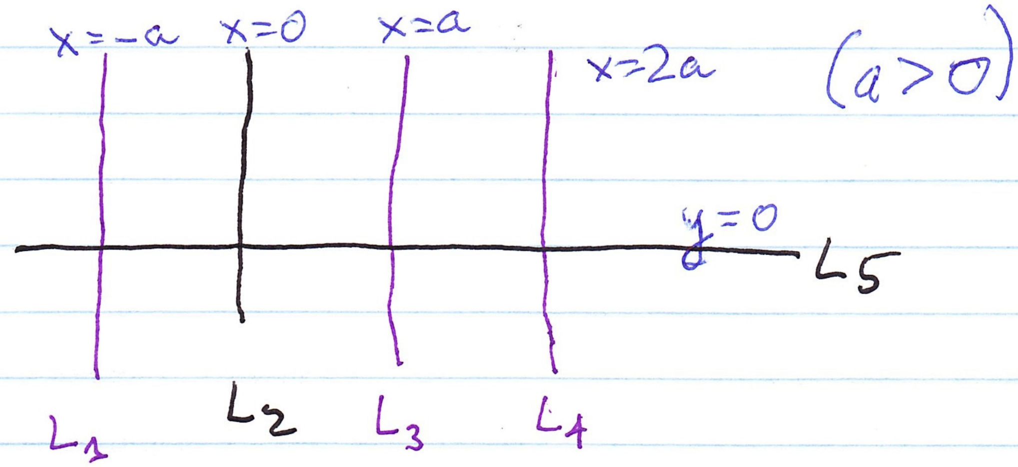
Solution to Problem 3 on p. 380 of Burton

The setup recommended in the problem is to choose coordinate axes to be L_2 (vertical) and L_5 (horizontal)



Suppose that $P=(x,y)$ is a point that lies on none of these lines. Then

$$p_1 = \text{distance}(P, L_1) = |x+a|$$
$$p_2 = \text{distance}(P, L_2) = |x|$$
$$p_3 = \text{distance}(P, L_3) = |x-a|$$
$$p_4 = \text{distance}(P, L_4) = |x-2a|$$
$$p_5 = \text{distance}(P, L_5) = |y|.$$

Then the defining equation for the locus is

$p_1 p_3 p_4 = a\, p_2 p_5$, which translates to

$$|x+a|\cdot|a-x|\cdot|2a-x| = a|x|\cdot|y|$$

which translates into a PAIR of equations

$$(x+a)(a-x)(2a-x) = \pm axy.$$

Note that is conclusion is not the same as the assertion in Burton. In fact, the locus is equal to the union of the curve

$$(x+a)(a-x)(2a-x) = axy$$

mentioned in Burton together with the mirror image of this curve with respect to the x-axis.

Analogy $|y| = |x|$ defines the union of the line $y=x$ with its mirror image: