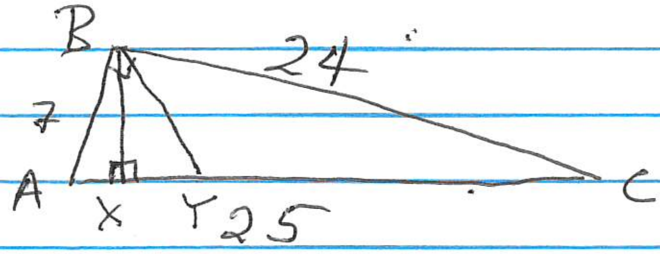
OUTLINES OF SOLUTIONS FOR

aabUpdate11598.pdf

1. 

Let $a = |BC|$, $b = |AC|$, $c = |AB|$, $h = |BX|$.

(i) Find $|AX|$. Since $|BX|$ is the mean proportional to $|AX|$ and $|XC|$ we have $h^2 = |AX|(25 - |AX|)$. By the Pythagorean Theorem we have $49 = 7^2 = |AX|^2 + h^2$. Combining these, we get

$$49 - |AX|^2 = |AX|(25 - |AX|).$$

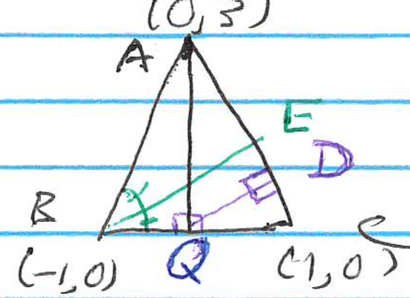
Now solve this for $|AX|$ and find $|CX| = 25 - |AX|$ $\blacksquare$

(ii) Find $|AY|$. By a result from III.5,

$$\frac{7}{24} = \frac{|AB|}{|BC|} = \frac{|AY|}{25 - |AY|}, \text{ so}$$

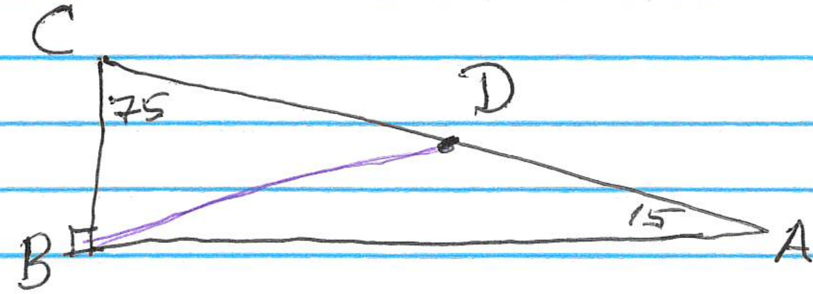
$175 - 7|AY| = |AY| \cdot 24$. Now solve for $|AY|$ $\blacksquare$

2. Same ideas applied to the figure below

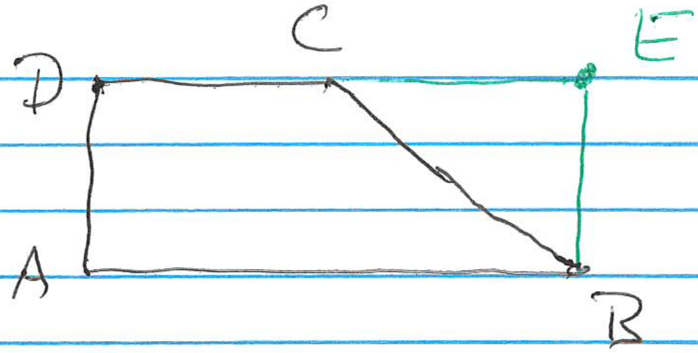


∎

3.



We only need D, really. By a result on semicircles, we know that A, B, C lie on a semicircular arc centered at D, so $|DA| = |DB| = |DC|$. Therefore the Isosceles Triangle Theorem implies $|\measuredangle DBC| = 75^\circ$. Since the angle sum of $\triangle DBC$ is 180° this means $|\measuredangle ADB| = |\measuredangle CBD| + |\measuredangle DCB| = 75^\circ + 75^\circ = 150^\circ$. ∎

4. 

Locate $E \in (DC$ so $|DE| = |AB|$, and note that $D * C * E$ because $|DE| > |DC|$.

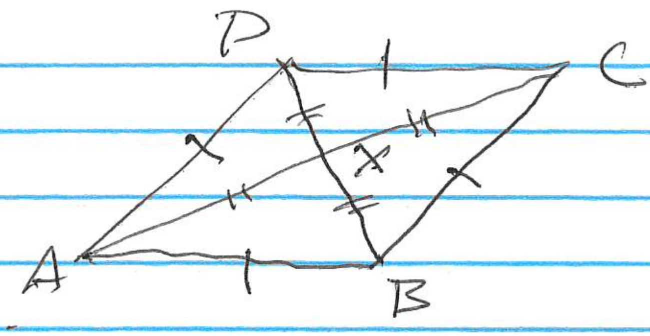
By a theorem on parallelograms, we know A, B, E, D are the vertices of one because $|AB| = |DE|$ and $AB \parallel DE$ (both $\perp AD$). Since there are right angles at D & A, this parallelogram must be a rectangle!

By the Exterior Angle Theorem,

$$|\angle DCB| > |\angle CEB| = 90°.$$

Since $\triangle BEC$ has a right angle at C we also know $|\angle CBA| = 90 - |\angle CBE| < 90$ since $|\angle CBE| > 0$. ■

5.



Since $|DA| = |DC|$ and $|BA| = |BC|$ it follows that BD is the perpendicular bisector of $[AC]$. Likewise AC is the perpendicular bisector of $[BD]$.

Therefore we have (by SSS) $\triangle XAB \cong \triangle XBC \cong \triangle XCD \cong \triangle XDA$, where the right angle is at X in each case and the edges passing through X have equal length. By the Isosceles Triangle Theorem and (angle sum $= 180^\circ$), we have

$$|\angle XAB| = |\angle XBA| = |\angle XBC| = |\angle XCB| =$$
$$|\angle XCD| = |\angle XDC| = |\angle XDA| = |\angle XAD| = 45^\circ.$$

Therefore
$$|\angle DAB| = |\angle XAD| + |\angle XAB| = 90^\circ$$
$$|\angle ABC| = |\angle XBA| + |\angle XBC| = 90^\circ$$

As in #4, this suffices to prove $ABCD$ is a rectangle. ∎