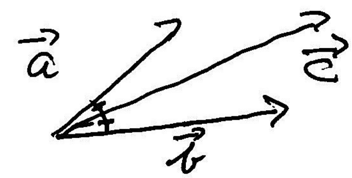
PROBLEM: $\vec{a}, \vec{b}$ nonzero vectors in $\mathbb{R}^2$, $\vec{c} = |\vec{b}|\vec{a} + |\vec{a}|\cdot\vec{b}$ (assume $\vec{a}, \vec{b}$ lin indep, so that $\vec{c} \neq 0$). To prove: The ray from $\vec{O}$ to $\vec{c}$ bisects $\measuredangle \vec{a}\vec{O}\vec{b}$.



SOLUTION: It is enough to show that $\cos\measuredangle \vec{c}\vec{O}\vec{a} = \cos\measuredangle \vec{c}\vec{O}\vec{b}$. But $\cos\measuredangle \vec{a}\vec{O}\vec{c} =$

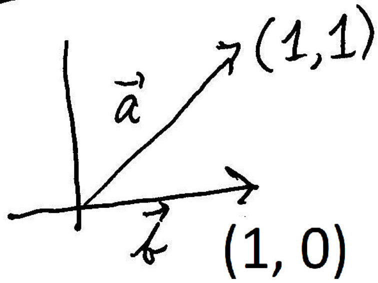
$$\frac{\langle \vec{a}, |\vec{a}|\vec{b} + |\vec{b}|\vec{a}\rangle}{|\vec{a}|\cdot|\vec{c}|} = \frac{|\vec{a}|\langle \vec{a}, \vec{b}\rangle + |\vec{b}||\vec{a}|^2}{|\vec{a}|\cdot|\vec{c}|} =$$

$$\frac{\langle \vec{a}, \vec{b}\rangle + |\vec{a}|\cdot|\vec{b}|}{|\vec{c}|}.$$

If we replace $\vec{a}$ with $\vec{b}$, interchanging the roles of $\vec{a}$ & $\vec{b}$, we get the same answer. ∎

EXAMPLES

1.



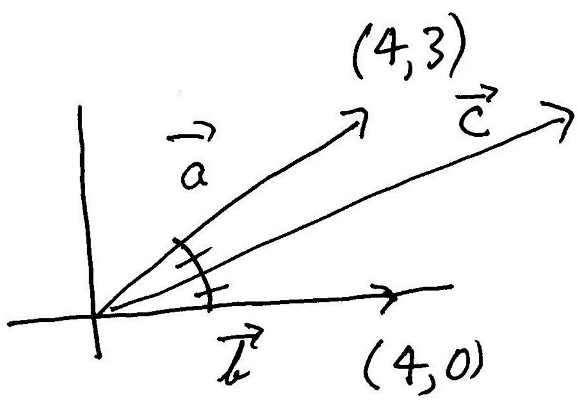
Say $\vec{a} = (1,1)$
$\vec{b} = (1, 0)$

Find $\vec{c}$.

SOLUTION $|\vec{a}| = \sqrt{2}$ & $|\vec{b}| = 1$, so the vector $\vec{c}$ is given by

$$\sqrt{2}\,(1, 0) + (1, 1) = (\sqrt{2} + 1, 2).$$

2.



$\vec{a} = (4,3)$
$\vec{b} = (4,0)$
$|\vec{a}| = 5, \ |\vec{b}| = 4$

$$\vec{c} = 4(4,3) + 5(4,0) = (36, 12)$$