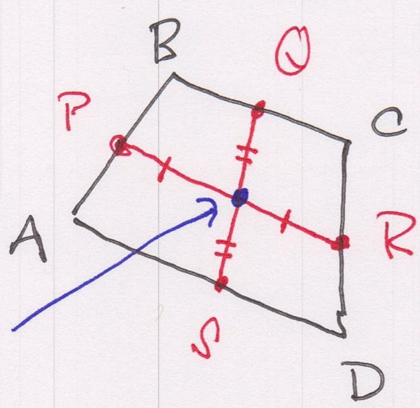
PROBLEM: Let A, B, C, D be four points in $\mathbb{R}^2$, no 3 of which are collinear, and let P, Q, R, S be the respective midpoints of $\{A,B\}$, $\{B,C\}$, $\{C,D\}$ and $\{A,D\}$. Prove that PR and QS meet, and the intersection is the midpoint of $\{P,R\}$ and $\{Q,S\}$.

IDEA:

To prove: This is the common midpoint



Show that $\frac{1}{2}(P+R) = \frac{1}{2}(Q+S)$.

SOLUTION: Express P, Q, R, S in terms of A, B, C, D: $P = \frac{1}{2}(A+B)$, $Q = \frac{1}{2}(B+C)$, $R = \frac{1}{2}(C+D)$, $S = \frac{1}{2}(A+D)$. Then we have

$$\frac{1}{2}(P+R) = \frac{1}{4}(A+B+C+D) = \frac{1}{2}(Q+S). \blacksquare$$

NOTE: This even works if A, B, C, D are not coplanar!!