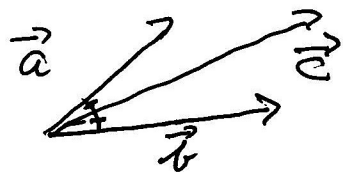


PROBLEM: \vec{a}, \vec{b} nonzero vectors in \mathbb{R}^2
 $\vec{c} = |\vec{b}|\vec{a} + |\vec{a}|\vec{b}$ (assume \vec{a}, \vec{b} lin indep, so
 that $\vec{c} \neq 0$). To prove: The ray from $\vec{0}$
 to \vec{c} bisects $\angle \vec{a} \vec{0} \vec{b}$.



SOLUTION: It is enough to show that
 $\cos \angle \vec{c} \vec{0} \vec{a} = \cos \angle \vec{c} \vec{0} \vec{b}$. But $\cos \angle \vec{a} \vec{0} \vec{c} =$

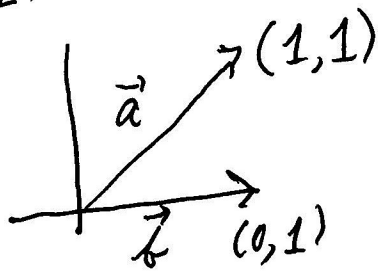
$$\frac{\langle \vec{a}, |\vec{a}|\vec{b} + |\vec{b}|\vec{a} \rangle}{|\vec{a}| \cdot |\vec{c}|} = \frac{|\vec{a}| \langle \vec{a}, \vec{b} \rangle + |\vec{b}| |\vec{a}|^2}{|\vec{a}| \cdot |\vec{c}|} =$$

$$\frac{\langle \vec{a}, \vec{b} \rangle + |\vec{a}| \cdot |\vec{b}|}{|\vec{c}|}. \text{ If we replace}$$

\vec{a} with \vec{b} , interchanging the roles of \vec{a} & \vec{b} ,
 we get the same answer. ■

EXAMPLES

1.



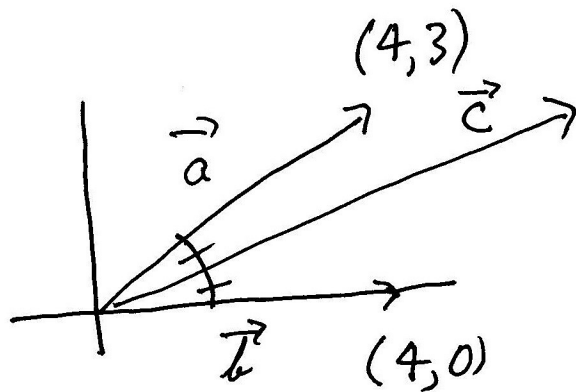
Say $\vec{a} = (1, 1)$
 $\vec{b} = (0, 1)$

Find \vec{c} .

SOLUTION $|\vec{a}| = \sqrt{2}$ & $|\vec{b}| = 1$, so the vector \vec{c} is given by

$$\sqrt{2}(0, 1) + (1, 1) = (\sqrt{2} + 1, 2).$$

2.



$\vec{a} = (4, 3)$
 $\vec{b} = (4, 0)$
 $|\vec{a}| = 5, |\vec{b}| = 4$

$$\vec{c} = 4(4, 3) + 5(4, 0) = (36, 12)$$