Alternate approach to the result in **betweenness.pdf**. We have $y_i = t_i x + (1 - t_i)z$ for $i = 1, 2, 3$. If we put the t_i in order there are six cases corresponding to the permutations of $\{1, 2, 3\}$. Look at each of these individually and see that $y_1 * y_2 * y_3$ is true if $t_1 < t_2 < t_3$ or vice versa, and the betweenness statement is false in the remaining four cases.

(B1) *is valid in the coordinate plane.* Let t_p be such that $p = b + t_p(d - b)$; to define the points a, c, e take $t_a = -1$, $t_c = \frac{1}{2}$ and $t_c = 2$.

(B2) *is valid in the coordinate plane.* Same notation as above without the specific values for t_a, t_b, t_c. Put t_a, t_b, t_c in order to see which point is between the other two.

Alternate proof of Proposition 4. We are given $a * b * d$ and $b * c * d$. Write $p = a + t_p(b - a)$ as before. Then $t_a = 0$, $t_b = 1$ and the hypotheses can be rewritten as

(1) Either $0 < 1 < t_d$ or else $t_d < 1 < 0$,

(2) Either $1 < t_c < t_d$ or else $t_d < t_c < 1$.

We can rule out the second option in (1) because $1 < 0$ is false, so that $0 < 1 < t_d$. The only option in (2) consistent with the latter is $1 < t_c < t_d$, so that $0 < 1 < t_c < t_d$. But these imply that $a * b * c$ and $b * c * d$ are true.

The next one was not done in class, but it is similar and might shed some light on the method in the previous paragraph.

Alternate proof of Example 3, *page* 49. Same notational conventions as in the preceding discussion; in particular, we have $t_a = 0$ and $t_b = 1$. Then $a * b * c$ means that $0 < 1 < t_c$ or else $t_c < 1 < 0$, so the former must be true. Also, $b * x * c$ means that $1 < t_x < t_c$ or else $t_c < t_x < 1$, and by the previous sentence this shows that $1 < t_x < t_c$ must be true. Therefore $0 < t_x < t_c$, so that $a * x * c$ must be true.

For the sake of completeness, here is one more.

Alternate proof of Example 2, *page* 49. Much as before, define t_x by $x = b + t_x(c - b)$. Since $a * b * c$ is true, the same kind of reasoning as before means that $t_a < 0 = t_b < 1 = t_c$. If $p \in [bc$ then $t_p \geq 0$, and if equality holds, then $p = b$; and in this case we know $p = b \in [ac$, so suppose that $t_p > 0$. But then we have $t_a < 0$ while t_p and $t_c = 1$ are both positive, so it follows that either $a * p * c$ is true or $a * c * p$ is true. In either case $p \in [ac$, and hence we have shown that $[bc \subset [ac$.