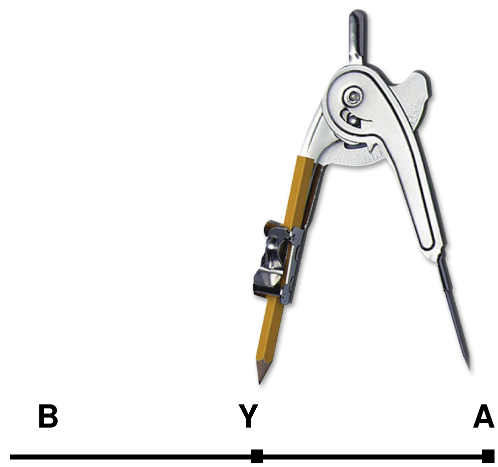
The compass property

This document gives an alternate proof of the following result from the notes:

<u>Proposition II.3.1.</u> *Let* **A** *and* **B** *be distinct points, and let* x *be a positive real number. Then there is a unique point* **Y** *on the open ray* **(AB** *such that* d(**A, Y**) $= x$. *Furthermore, we have* **A**∗**Y**∗**B** *if and only if* $x < d$(**A, B**), *and similarly we have* **A**∗**B**∗**Y** *if and only if* $x > d$(**A, B**).



<u>Proof.</u> We know there is a ruler function from **AB** to the real line such that the real numbers a, b corresponding to **A, B** satisfy $a < b$. If we choose the point **Y** which corresponds to $a + x$, then it follows that d(**A, Y**) $= x$. Furthermore, **Y** lies on the open ray because we have **A**∗**Y**∗**B** if $x < b - a$, **Y** = **B** if $x = b - a$, and **A**∗**B**∗**Y** if $x < b - a$. This shows existence.

To prove uniqueness, suppose that **Z** is a point on the open ray such that d(**A, Z**) $= x$; let z correspond to **Z** with respect to the given ruler function. Since **Z** lies on the open ray **(AB**, it does not lie on the opposite ray, so **Z**∗**A**∗**B** is false and therefore $z < a < b$ is also false. Therefore z is not less than a, and since **Z** and **A** are distinct it also follows that z is not equal to a. It follows that $z > a$. Now the conditions $|z - a| = d$(**A, Z**) $= x$ and $z > a$ imply that $z = x + a$; since the latter corresponds to **Y** by construction, we have **Y** = **Z**.■