HYPERPLANE SEPARATION AND VECTOR ALGEBRA

This is a generalization of the concepts and results in `separation.pdf`.

The main result is the following: *If $P \subset \mathbb{R}^n$ is a hyperplane, then the set of points not on P is a union of two sets H_+ and H_- which are nonempty convex subsets such that if $x \in H_+$ and $y \in H_-$, then the open segment (xy) contains a point of P.*

The case $n = 3$ of this statement is the *space separation posbulate* in the notes. The proof of this result proceeds as in the prior document with $\mathbb{R}^n$ replacing $\mathbb{R}^2$ and the equation $\sum a_j x_j = b$ defining the hyperplane P. The two half spaces will be defined by $\sum a_j x_j > b$ and $\sum a_j x_j < b$.

A typical problem involving this property would involve a finite set of points $c_1, ..., c_i, ...$ with coordinates $c_{i,j}$. Determining which points lie on H_+, P and H_- amounts to determining whether the sums $\sum_j a_j c_{i,j}$ are greater than, equal to, or less than b.