Another example involving congruence axioms

Here is another example of an axiom model showing that the Congruence Axioms in Section **II.4** of the notes cannot be proved from the previously introduces sets of axioms.

Take the plane with the *maximum metric* $\boldsymbol{d_\infty}$, for which the distance between the points $\mathbf{x} = (x_1, x_2)$ and $\mathbf{y} = (y_1, y_2)$ is the larger of $|x_1 - y_1|$ and $|x_2 - y_2|$. Then the plane with this notion of distance satisfies the axioms for linear measurement, and if we add the usual notion of angular measurement then we have a system which satisfies all the axioms through Section **II.3.** Here is an example to show that **SAS** is false in this system. Take the triangle whose vertices are $(0, 0)$, $(1, 0)$ and $(0, 1)$. Then the distances between each pair of distinct vertices is 1. If **SAS** is true, then this equilateral triangle is equiangular. However, angle measurement in this model is the usual one, so we know that the given triangle is not equiangular, and therefore we may conclude that **SAS** is false in our system.■