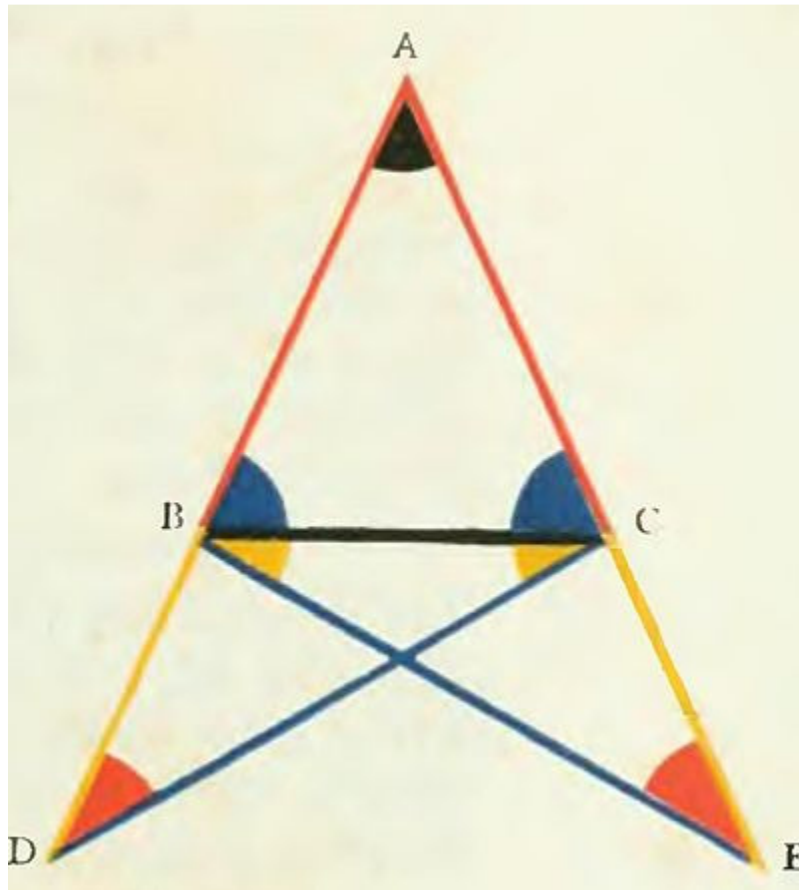


## Note on the Isosceles Triangle Theorem

The proof given in the notes is attributed to Pappus of Alexandria from the 4<sup>th</sup> century A.D.. Euclid's proof of the "only if" implication in the *Elements* is considerably more complicated, and the theorem with this proof is known as *Pons Asinorum*. There is an extensive discussion of the subject in [https://en.wikipedia.org/wiki/Pons\\_asinorum](https://en.wikipedia.org/wiki/Pons_asinorum). The name probably arises from the main drawing for the proof, the bottom of which may be viewed as resembling a bridge:



The idea of Euclid's proof is to choose **D** and **E** such that  $|BD| = |CE|$ , which implies that triangles **ABE** and **ACD** are congruent in the sense of the course notes. This means that the angles colored in orange have equal measures, which in turn implies that triangles **DBC** and **ECB** are congruent. The two congruences imply that angles **ACD** and **ABE** have equal measures, and likewise angles **BCD** and **CBE** have equal measures. By the additivity property of angle measures, it follows that angles **ABC** and **ACB** have equal measures, which is one implication direction of the proof.