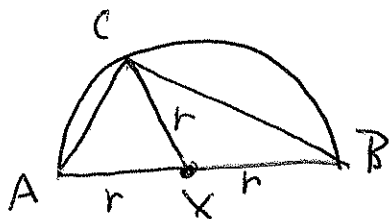


THEOREM. If  $\triangle ACB$  is inscribed in a semicircle, it is a right angle.



$|\angle TUV| = \text{measure of } \angle TUV \text{ in degrees}$

Let  $X$  be the center of the circle, so segments  $[AX]$ ,  $[XB]$  and  $[CX]$  have length  $r$ .

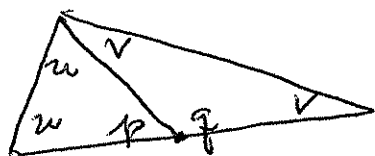
By Isosceles Triangles

$$|\angle XAC| = |\angle XCA|; \text{ call it } u.$$

$$|\angle XBC| = |\angle XCB|; \text{ call it } v.$$

$$\text{Let } p = |\angle CXA|, q = |\angle CXB|.$$

Since the angle sum of a triangle is  $180^\circ$ , we have



$$2u + p = 180^\circ$$

$$2v + q = 180^\circ$$

$$\text{Also, } p + q = 180^\circ.$$

Let's manipulate these equations.

Add the first two:

$$2u + 2v + p + q = 360^\circ$$

Subtract the third from this:

$$2u + 2v = 180^\circ$$

Divide by 2:

$$u + v = 90^\circ$$

Since  $\angle ACB = u + v$ , it follows that  $\angle ACB$  is a right angle.

According to <sup>later</sup> sources such as Herodotus and Proclus, this result was known to Thales, but there are no <sup>known</sup> primary sources.