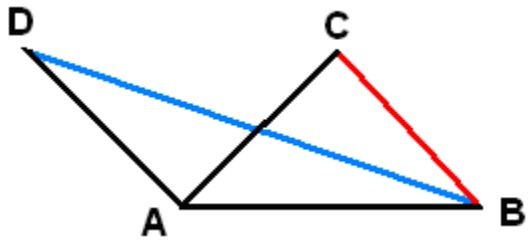
The Hinge Theorem

This is one of several inequality theorems for triangles**.**

Theorem. *Given triangles* △**ABC** *and* △**ABD** *which satisfy* |**AC**| = |**AD**|**,** *then we have* |**BC**| < |**BD**| *if and only if* |∠**CAB**| < |∠**DAB**|**.**



This result**,** which can be proved using the Law of Cosines from trigonometry

$$c^2 = a^2 + b^2 - 2ab\cos|\angle \mathbf{ACB}|$$

yields a companion result to the Pythagorean Theorem.

Corollary. *Given triangle* △**ABC,** *let* a = |**BC**|, b = |**AC**|, *and* c = |**AB**|. *Then* ∠**CAB** *is acute if* $c^2 < a^2 + b^2$ *and* ∠**CAB** *is obtuse if* $c^2 > a^2 + b^2$**.**