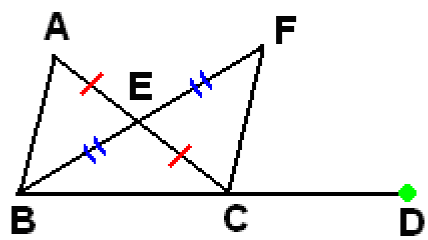
The angle sum of a triangle

This is a slightly shorter alternate proof of the following basic result:

<u>Theorem III.2.13.</u> *Given* $\triangle$**ABC** *in a Euclidean plane, we have the equation*

$$|\angle ABC| + |\angle BCA| + |\angle CAB| = 180°.$$

<u>Proof.</u> We shall prove this using the setting of the Exterior Angle Theorem (= Theorem **III.2.1**). In the proof of the later result, **D** is a point such that **B**∗**C**∗**D**, while **E** is the midpoint of **[AC]** and **F** is chosen such that **E** is the midpoint of **[BF].** The proof then shows that $|\angle BAE| = |\angle ECF|$ and **F** lies in the interior of $\angle ACD$.



Since we know that **B**∗**E**∗**F** and **A**∗**E**∗**C,** it follows that **B** and **F** lie on opposite sides of the line **AC,** so that $|\angle BAE|$ and $|\angle ECF|$ are alternate interior angles for the transversal **AC.** If we combine this with $|\angle BAE| = |\angle ECF|$, we can use Proposition **III.2.10** to conclude that **AB** || **CF.** But the betweenness conditions at the beginning of this paragraph also imply that **A** and **F** are on the same side of **BC,** and therefore by Corollary **III.2.12** and **AB** || **CF** it follows that $|\angle ABC| = |\angle FCD|$ (the two angles in this equation are corresponding angles for the transversal **BC**).

Since **F** lies in the interior of $\angle ACD$, we have $|\angle ACD| = |\angle ACF| + |\angle FCD|$; also, since $|\angle ACF| = |\angle BAC|$ and $|\angle FCD| = |\angle ABC|$, we may rewrite the equation in the first part of this sentence as $|\angle ACD| = |\angle BAC| + |\angle ABC|$. But by the Supplement Postulate we also have $|\angle ACB| + |\angle ACD| = 180°$, and if we combine this with the previous equation we obtain the desired formula

$$|\angle ABC| + |\angle BAC| + |\angle ACB| = 180°. \blacksquare$$