The Poincaré metric for hyperbolic geometry

In the Poincaré model for hyperbolic geometry, it is often illuminating (and sometimes necessary) to view the distance function by giving a formula for finding the lengths of arbitrary "nice" curves and noting that the "lines" in hyperbolic geometry will be the shortest curves joining the endpoints. Fundamental examples of "nice" curves are parametrized curves $\gamma(t) = (x_1(t), \cdots, x_n(t))$ in the disk

$$\mathbb{H}^n \quad = \quad \{v \in \mathbb{R}^n \mid |v| \ < \ 1\}$$

which can be split into finitely many pieces on which the coordinate functions all have continuous derivatives. This class is broad enough to contain all circular arcs and straight line segments in the disk $\mathbb{H}^n$. If we consider the piece of γ for which $t \in [a, b]$, then the hyperbolic arc length of this piece with respect to the Poincaré metric is equal to

$$\int_a^b \sqrt{\frac{4 \cdot \sum_i x_i'(t)^2}{1 - \sum_i x_i'(t)^2}} \, dt \ .$$