**Math 133, Fall 2018, Quiz 1**

In the coordinate plane, find the barycentric coordinates of $P = (4, 2)$ with respect to the points $A = (1, 0)$, $B = (6, 5)$ and $C = (6, -5)$.

**Solution.** The first step is to subtract $A$ from $B$, $C$ and $P$:

$$B - A \;=\; (5, 5)\,, \quad C - A \;=\; (5, -5)\,, \quad P - A \;=\; (-3, 2)$$

Next, express $P - A$ as a linear combination of $B - A$ and $C - A$. We need to find $x$ and $y$ so that $P - A = x(B - A) + y(C - A)$:

$$(-3, 2) \;=\; x(5, 5) \;+\; y(5, -5) \;=\; (5x + 5y, 5x - 5y)$$

The solution of this system is given by $x = -1/10$ and $y = -1/2$; details are omitted. Therefore we have

$$P - A \;=\; -\frac{1}{10}(B - A) \;-\; \frac{1}{2}(C - A)$$

which can be rewritten as follows:

$$P \;=\; \frac{8}{5}A \;-\; \frac{1}{10}B \;-\; \frac{1}{2}C$$

Note that the coefficients of the three reference vectors add up to 1, so the coefficients give the barycentric coordinates.∎