

## Exercises

1. Suppose  $x, y, z \in L$  are distinct. Prove that  $L = xy = xz = yz$ . [Useful identity in more substantial proofs.]

3. More complicated identities like #1  $A \neq B$   
 (a) If  $X \in (AB)$ , then  $(AX) = (AB)$ .  
 (b) If  $C \in A * B$ , then  $(AC) = (AB)^{op}$ .

Hint for (b): If  $f: AB \rightarrow \mathbb{R}$  ruler fun. with  $f(B) > f(A)$ , then  $-f$  satisfies  $-f(C) > -f(A)$ .

2. If  $A \neq B$ , prove that  $(AB)^{op}$  consists of all  $X$  such that  $X * A \neq B$ .

4. Verify the following using ruler functions

If  $X \neq Y$  in  $\left\{ \begin{array}{l} [AB] \\ (AB) \\ [AB]^{op} \end{array} \right\}$  and  $X * Z \neq Y$ , then also.

$Z \in \left\{ \begin{array}{l} [AB] \\ (AB) \\ [AB] \end{array} \right\}$ .