Math 133
Fall 2021

MORE EXERCISES FOR WEEK 01

The following exercises deal with consequences of the Incidence Axioms and the Ruler Postulate. Assume that $(\mathbf{S};\mathcal{P};\mathcal{L};d)$ or $(\mathbf{P};\mathcal{L};d)$ is a system which satisfies these axioms.

5. Suppose that $A \neq B$; if AB is the line joining A to B, prove that $AB = [AB \cup [BA$.

6. Suppose that $A * B * C$; prove that $[AB = [AB] \cup [BC$.

7. Let P be a plane, let $\mathbf{a}, \mathbf{b}, \mathbf{c}$ be noncollinear points on P, let $\mathbf{z}$ be a point which is not on P, and let $\mathbf{u}$ and $\mathbf{v}$ be distinct points on the line $\mathbf{cz}$ satisfying $\mathbf{u}, \mathbf{v} \neq \mathbf{c}$. Show that the planes P, $\mathbf{abu}$ and $\mathbf{abv}$ are distinct.

8. If P_1, P_2 and P_3 are distinct planes in a 3-space $\mathbf{S}$, show that their intersection is either the empty set, a single point or a line. Describe examples in ordinary geometry for which the intersections are of each (mutually exclusive) type.

9. Suppose that A, B, C, D are four noncoplanar points in a 3 - space $\mathbf{S}$. Explain why the lines AB and CD are disjoint but not coplanar (in other words, they form a pair of *skew lines*).

10. Let L be a line in a 3-space $\mathbf{S}$, and let X be a point which does not lie on L. Using the Incidence Axioms, prove that there is a unique plane P such that $X \in P$ and $L \subset P$. [*Hint:* If A and B are two points of L, why are A, B and X noncollinear?.]

11. Suppose that we are given three distinct lines L, M, N in a 3-space $\mathbf{S}$ such that (i) the lines all contain some point X, (ii) each of the lines has a point in common with a fourth line K which does not contain X. Prove that there is a plane containing all four lines. [*Hint:* Draw a sketch of the situation to get some insight.]

12. Suppose that we are given points A, B, C and X, Y, Z in a plane or 3-space such that $A*B*C$ and $X * Y * Z$ both hold, and in addition we have $d(A, C) = d(X; Z)$ and $d(B, C) = d(Y, Z)$. Prove that $d(A, B) = d(X, Y)$. [*If equals are subtracted from equals, the differences are equal.*]