

Solutions to exercises

1. L is the unique line containing $x+y$, $x+z$ and $y+z$. Hence we may write

$$L = xy = xz = yz. \blacksquare$$

2. Since three collinear points \Rightarrow exactly one between other two, if $X \in L = AB$ then exactly one of $X=A$, $X=B$, $A * X * B$, $A * B * X$, $X * A * B$ is true. By definition $X \in [AB] \Leftrightarrow$ one of the first four is true. Since $(AB)^{op} \Leftrightarrow X \in L$ but $X \in [AB]$, we are forced to conclude that $X \in (AB)^{op} \Leftrightarrow X * A * B$ is true. \blacksquare

3. (a) Choose a ruler function so that

$$f(B) > 0 = f(A). \text{ Then } X \in (AB) \Leftrightarrow$$

$f(X) > 0$. By the description of (AX) , it is all Y so that $f(Y) > 0$, and this is precisely the definition of (AB) . \blacksquare

3 (A) Choose f as before, let $g = -f$.
Then $f(C) < 0 < f(A)$, so $(AB)^{op} =$

all X st. $f(X) < 0 =$ all X st. $g(X) > 0$.

Since $g(C) > 0$ this means $C \in$

$$X \in (AB)^{op} \iff f(X) < 0 \iff g(X) > 0 \iff X \in (AC)$$

4. Case (a) Set is $[AB]$. Then $0 < f(X), f(Y) < f(B)$. Two subcases (i) $f(X) < f(Y)$. Then

$$X * Z * Y \implies f(X) < f(Z) < f(Y), \text{ so}$$

$$f(A) \leq f(X) < f(Z) < f(Y) \leq f(B). \quad \square$$

(ii) $f(Y) < f(X)$. Switch roles of X & Y in the preceding argument.

Case (b) Same idea but now $f(A) <$ both $f(X)$ & $f(Y)$ and $f(B) >$ both.

Case (c) Set is $[AB]$, so $f(X), f(Y) \geq 0$.

Two subcases as before: (i) $f(X) < f(Y) \implies$

$$f(A) \leq f(X) < f(Z) < f(Y) \implies Z \in [AB]. \quad \text{(ii)}$$

Switch X & Y to conclude the reverse case. \square

Note Similar conclusions true for (AB) ,

$[AB]^{op}$, $(AB)^{op}$.