Math 133
Fall 2021

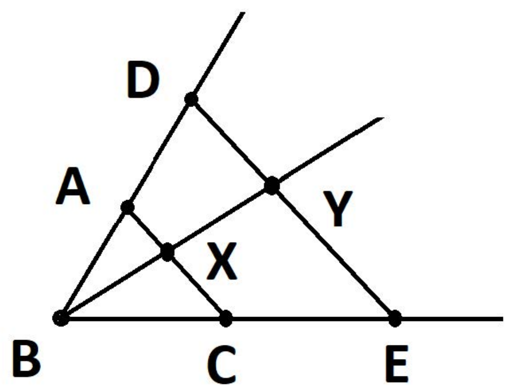
MORE SOLUTIONS FOR WEEK 02 EXERCISES

Assume that $(\mathbf{S}; \mathcal{P}; \mathcal{L}; d; \alpha)$ or $(\mathbf{P}; \mathcal{L}; d; \alpha)$ is a system which satisfies the Incidence, Ruler, Plane or Space Separation (as appropriate), Angle Measurement and Triangle Congruence Axioms.

11. Assume that L contains points in each of (AB), (AC) and (BC). Since L contains a point in each of (AB), (AC) then both B and C lie on the side of L opposite A. By convexity the side containing B and C also includes all points of (BC). Therefore L cannot contain a point of (BC), which contradicts the first sentence. Therefore L cannot contain points in each of (AB), (AC) and (BC).■

12. The midpoint conditions implies that $|DA| = \frac{1}{2}|AC| = \frac{1}{2}|BC| = |EB|$. If we combine this with the tribial identity $|AB| = |BA|$ and the isosceles triangle hypothesis, we conclude that $\triangle DAB \cong \triangle EBA$ by SSS.■

13. Here is a drawing which reflects the hypotheses.



By constuction we have $D \in (BA$ and $E \in (BC$, and since $A * X * C$ we also have $X \in \text{Int } \angle ABC$. Therefore the Crossbar Theorem implies that there is a point $Y \in (ED) \cap (BX$. It remains to verify that $B * X * Y$.

We shall do this by eliminating the other possibilities, which are $B * Y * X$ and $X = Y$. To see that $X \neq Y$, observe that $B * A * D$ and $B * C * E$ imply that A, B, C all lie on the same side of DE; since $A * X * C$, by convexity we also know that X lies on this side of DE. Therefore $X \notin DE$ but $Y \in DE$, so that $X \neq Y$.

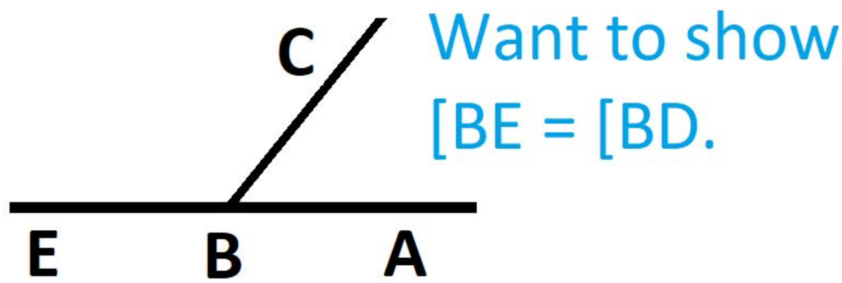
If $B * Y * X$ we true, then B and X would be on opposite sides of DE. However, this contradicts the conclusions in the preceding paragraph, and hence we must have $B * X * Y$.■

14. By Exercise 12 we have $\triangle ABD \cong \triangle ACD$, and also $D \in \text{Int } \angle BAC$. This implies $|\angle BAD| = |\angle DAC|$ and by the Additivity Postulate for angle meaures we also have

$$|\angle BAC| = |\angle BAD| + |\angle DAC| = 2|\angle BAD| = 2|\angle DAC| .$$

If we multiply each of these equations by $\frac{1}{2}$ we see that $|\angle BAD| = |\angle DAC| = \frac{1}{2}|\angle BAC|$.■

15. Let E satisfy $A * B * E$; by the Supplement Postulate we have $|\angle ABC| = 180° - |\angle EBC|$, and we also know that neither D nor E lies on the same side of BC as A (also $E \notin BC$).



By the Protractor Postulate there is only one ray $[BX$ such that $(BX \subset \{D, E\}$ − side BC and $|\angle XBC| = 180° - |\angle ABC|$. Since $[BD$ and $[BE$ both have this property, these rays must be equal. This means that D lies on the line containing A, B and E, or equivalently E lies on the line containing A, B and D. Finally, since $(BE$ is the set of all Y such that $A * B * Y$ and $[BD$ and $[BE$ it follows that $A * B * D$ holds.■