

Euclidean parallelism

Def. Two lines L & M ($L \neq M$) are parallel if they are coplanar and disjoint. $L \parallel M$

Example $X \notin L$ Let K be \perp to L through X , $M =$ perpendicular to K through X .

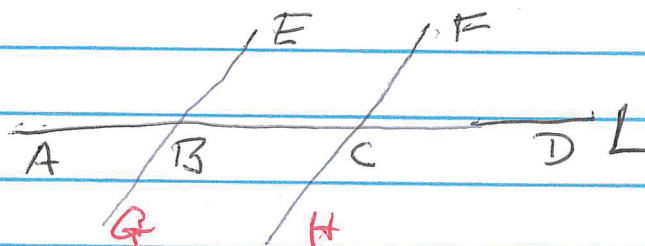
Then $L \parallel M$, for if $Y \in L \cap M$ then

(1) $Y \notin K$; otherwise $Y \in K \cap L \cap M$ so

both L and M are perpendicular to K at Y CONTRA
DICTION

(2) there is only one \perp to K through Y .

Even more is true.



E, F same side L .

If $|\angle EBC| = |\angle EBD| = |\angle FCD|$, then $BE \parallel CF$.

Exclude all options Suppose $X \in BE \cap CF$.

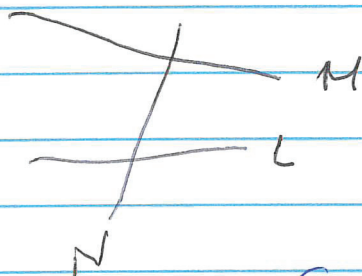
$X \notin L$. Would imply $X = B = C$. Since $X \in L \cap BE$ & $X \in L \cap CF$.

$X \in E \neq F$ side of L , for then the consequences of E.A.T. would imply $\angle FED > \angle EBC$.

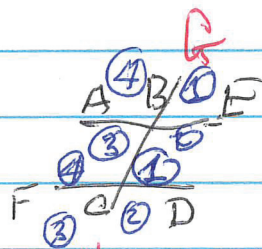
$X \in$ opposite side, for E.A.T. implies $\angle FED = 180 - \angle HED = 180 - \angle GBC = \angle EBC$.

Transversals

$L \neq M$ coplanar. Transversal N must both intersect, not the point in $L \cap M$.



Angle pairs



$A + B + E, D + C + F$

$A + F$ on same side BD
 $D + E$ on same side BD
 $A + D$ on opp sides.

alternate interior angles $\angle ABC + \angle BCD, \angle EBC + \angle FED$

corresponding angles (1) etc.

consecutive angles $\angle BCD + \angle CBF$ etc.

alternate exterior angles Two pairs

Half of H.S. Geometry Theorem

In prev. setting, each of the following implies $AB \parallel CD$.

(1) A pair of corresponding angles have = measures.

(2) Likewise for alternate $\left\{ \begin{array}{l} \text{interior} \\ \text{exterior} \end{array} \right\}$ angles.

(3) A pair of consecutive angles is supplementary (measures add up to 180).

Proofs (1) is the previous result.

(2.1) Will only consider alt. int. pair $\angle ABC$ & $\angle BCD$. Other case follows by switching roles of letters.

Here we are given $|\angle ABC| = |\angle DEF|$. By Vert. \angle Thm., $|\angle GBE| = |\angle ABC| = |\angle DEF|$.
Apply (1).

(2.2) Will only consider $\angle ABG$ & $\angle HCD$ as in part 2.1. ~~Vert. \angle Thm.~~ Supp. Postulate \Rightarrow
 $|\angle ABG| + |\angle ABC| = 180 = |\angle HCD| + |\angle BCD|$.

We're given $|\angle ABG| = |\angle HCD|$, so $|\angle ABC| = |\angle DEF|$.

Now apply (2.1).

(3) Only consider one case as before.
 Now we're given $\cancel{1\angle ABC} + \overset{ERC}{1\angle BCD} = 180$.

Supp. Post. $\Rightarrow 1\angle ABC + 1\angle EBC = 180$.

Hence $1\angle ABC = 180 - 1\angle EBC = \overset{180}{1\angle BCD}$.

Again apply (2.1).

EUCLIDEAN PARALLELISM

"Fifth" (Parallel) Postulate Given line L

→ Assume
 until
 further
 notice!

and $X \notin L$ there is only one line M so $X \in M$ &
 $L \parallel M$. (Euclidean Parallel Postulate)

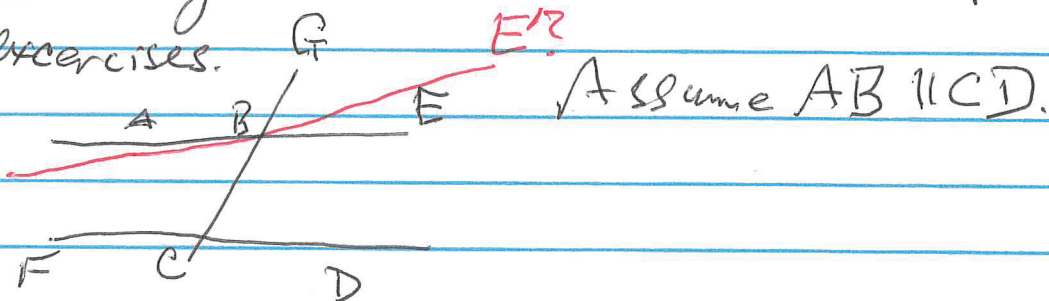
Immediate consequence L, M, N distinct coplanar
 lines. If $L \parallel M$ & $M \parallel N$, then $L \parallel N$.

Proof If not, then $Y \in L \cap N \Rightarrow L \& N$ $\parallel M$
 through Y .

(3D version also true but harder to prove). Must prove $L \& N$ coplanar!

Other half of H.S. Theorem. Prev. situation
 $AB \parallel CD \Rightarrow$ all of the listed sufficient
 conditions.

Proof Only show (1) holds. Others left at exercises.



Protractor Post, \Rightarrow unique ray $[BE'$ so $\angle CBE' \subseteq$
 $D+E$ side BC and $|\angle GBE'| = |\angle GCD|$.

Then AB & BE' are lines through B & $\parallel CD$.

Hence $AB = BE'$ and thus $[BE' = [BE$, so

$$|\angle ABE| = |\angle GBE'| = |\angle BCD|.$$

FILL IN THE REST!!

Cor. 1 Suppose $L, M, N \subseteq$ plane & $L \parallel N$,
 $L \perp M$. Then also $M \perp N$.

Proof Key point is N & M not parallel (else $L \parallel N$
 $\neq N \parallel M \Rightarrow L \parallel M$, which is false). Now use prev.
 results,

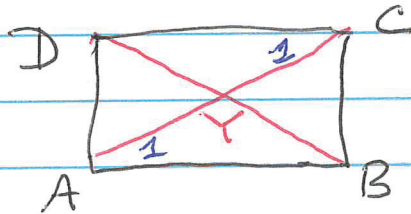
Cor. 2 $L \perp M$, $N \perp K$ and $L \parallel N$
 $\Rightarrow M \perp K$.

Proof $M \parallel K$ and $L \perp M \Rightarrow L \perp K$. But $L \perp K$ & $N \perp K$
 would $\Rightarrow L \parallel N$, contradiction.

Def. Say A, B, C, D form vertices of a rectangle if $AB \perp BC, BC \perp CD, CD \perp DA$ and $DA \perp AB$. Automatically convex.

Proposition. $|AB| = |CD|$ and $|AD| = |BC|$.

Proof.



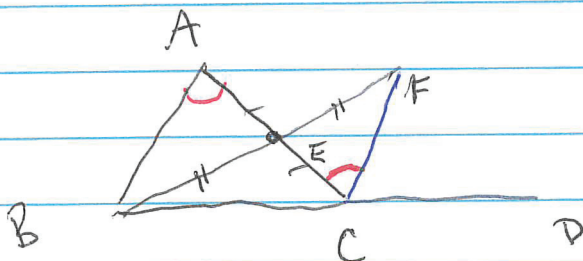
$AB \parallel CD$ & $BC \parallel AD$. Also (AC) & (BD) meet at same point Y . $B+D$ on opp sides $AC \Rightarrow$

$\angle DCA = \angle ACB$, A & C on opp sides BD also $\angle BCA = \angle DAC$. Hence $\triangle ADC \cong \triangle CBA$ by A.S.A., so $|AB| = |CD|$ and $|AD| = |BC|$.

Angle Sum of a Triangle

In $\triangle ABC$, $\angle BAC + \angle ABC + \angle ACB = 180$.

Proof. Use setting of Exterior Angle Thm.



Recall some conclusions

What more can we say if \parallel postulate is true?

$\angle BAC = \angle ACF$ so $BA \parallel CF$ by first half of thm on transversals. Now $ETP \Rightarrow$ for corresponding angles we have

$$\angle ABC = \angle FCD.$$

We saw $F \in \overline{ACD}$, so

$$\angle ACD = \angle ACF + \angle FCD = \angle BAC + \angle ABC.$$

$$\text{Hence } 180 = \angle ACB + \angle ACD =$$

$$\angle ACB + \angle BAC + \angle ABC.$$