Math 133
Fall 2021

MORE EXERCISES FOR WEEK 04

For the these exercises assume that $(\mathbf{S}; \mathcal{P}; \mathcal{L}; d; \alpha)$ or $(\mathbf{P}; \mathcal{L}; d; \alpha)$ is a system which satisfies the axioms for Euclidean geometry.

7. (a) Suppose that we are given real numbers $a, b > 0$. Prove that there is a right triangle $\triangle ABC$ with $|BC| = a$, $|AC| = b$ and $|\angle ACB| = 90°$.

(b) Using (a) and the Hinge Theorem (see Moise, p. 121), prove that in a triangle $\triangle ABC$ we have $|\angle ACB| < 90°$ when $|AB|^2 < |AC|^2 + |BC|^2$ and $|\angle ACB| > 90°$ when $|AB|^2 > |AC|^2 + |BC|^2$.

8. Suppose we are given isosceles triangle $\triangle ABC$ with $|AB| = |AC| = x$ and $|BC| = y$. Let $D \in (AC)$ be such that $[BD$ bisects $\angle ABC$, and let $z = |CD|$. Solve for z in terms of x and y.

9. Suppose we are given $\triangle ABC$, and let $D \in (AB)$ be such that $|\angle DCA| = |\angle ABC|$. Prove that $|AC|$ is the mean proportional between $|AB|$ and $|AD|$.

10. Suppose that we are given $\triangle ABC \sim \triangle DEF$, and let G and H be the midpoints of $[BC]$ and $[EF]$ respectively. Prove that $\triangle ABG \sim \triangle DEH$.

11. Suppose that $\triangle ABC \sim \triangle DEF$ and $\triangle ABC$ is isosceles. Show that $\triangle DEF$ is also isosceles.

12. Justify the assertion about planar coordinate systems: *If A, B, C are noncollinear points in the plane, then there is a coordinate system such that A corresponds to $(0, 0)$, B corresponds to $(u, 0)$ for some $u > 0$, and C corresponds to (x, y) where $y > 0$.*