Math 133
Fall 2021

EXERCISES FOR WEEK 7

For these exercises assume that the coordinate plane satisfies the axioms for Euclidean plane geometry, with distances and lines as described in Chapter 17 of Moise.

1. Construct a finite incidence planein which one line has three points and the remaining lines each have two points. Explain why this implies that neither the statement

> *for each pair of lines L and M, there is a* 1–1 *correspondence between the points of L and the points of M*

nor its negation is a consequence of the axioms for an incidence plane.

2. The axioms for an algebraic system called a *commutative ring*, which consists of a nonempty set together with notions of addition and multiplication, are given as follows:

(Commutative law of addition) $a + b = b + a$ for all a and b.

(Associative law of addition) $a + (b + c) = (a + b) + c$ for all a, b, c.

(Zero element) There is an element 0 in the system such that $a + 0 = a$ for all a.

(Existence of negatives) For each a there is an element a^* in the system such that $a + a^* = 0$.

(Commutative law of multiplication) $ab = ba$ for all a and b.

(Associative law of addition) $a(bc) = (ab)c$ for all a, b, c.

(Distributive law) $a(b + c) = ab + ac$ for all a, b, c.

Prove that the distributive law is not a consequence of the preceding axioms. [*Hint:* Define a new pair of operations on the integers, rationals or real numbers with the usual notion of addition and a new multiplication of the form $a \circ b = a + b + ab$.]