Math 133
Fall 2021

SOLUTIONS FOR “MORE WEEK 07 EXERCISES”

1. By definition the minor arc joining A and B consists of A, B and all points $X \in \Gamma$ which are on the opposite side of AB as Q. We need to show that the points of the second type are the same as the points of $\Gamma \cap \mathrm{Int}\,\angle AQB$.

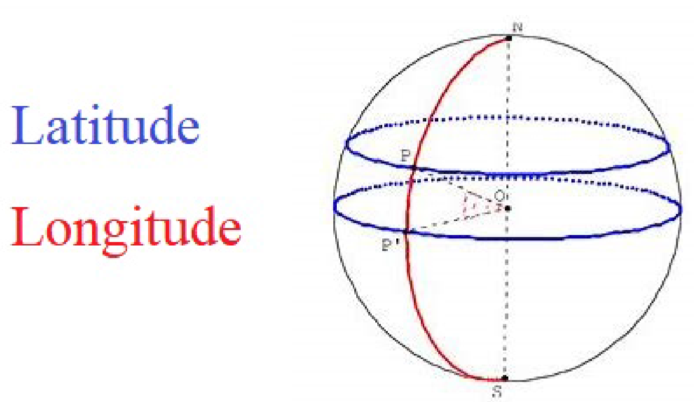
Suppose that X lies in the second set. By the Crossbar Theorem we know that there is a point $Y \in (QX \cap (AB)$. Therefore Exercise 9.2 implies that $|QY| < |QX|$ because the latter is the radius of the circle. Since $Y \in (QX$ it follows that $Q * Y * X$ and hence Q and X lie on opposite sides of AB.

Conversely, suppose that $X \in \Gamma$ and X lies on the side of AB opposite Q. By Plane Separation we then have a point $Y \in (QX) \cap AB$. Since $Q * Y * X$ implies $|QY| < |QX|$ we know that Y lies in the interior of Γ. We can now apply Exercise 9.2 to conclude that $Y \in (AB)$.■

2. The main results cited in the proofs are the theorem stating that the larger angle is opposite the longer side.the hypotenuse-side congruence theorem for right triangles, the Perpendicular Bisector and Isosceles Triangle Theorems, and the existence of unique perpendiculars to a line througha given point (within a plane). The proofs in this course for all of these theorems were done for neutral geometries.■

3. This is worked out in `solutions14a.pdf`.■

4. The statement is **FALSE.** By definition a great circle on a sphere is a circle whose center coincides with the center of the sphere. In particular, the length of a great circle is the circumference of the sphere. Neither of these is true for the latitude circle. For a sphere of radius r, the center of this circle is a point which is $r/\sqrt{2}$ from the center of the sphere, and the length of the latitude is $1/\sqrt{2}$ times the circumference of the sphere.



In the drawing above, the radius of the latitude through P is given by $r \cos \angle POP'$, where $|\angle POP'|$ is the latitude and r is the radius of the sphere.■