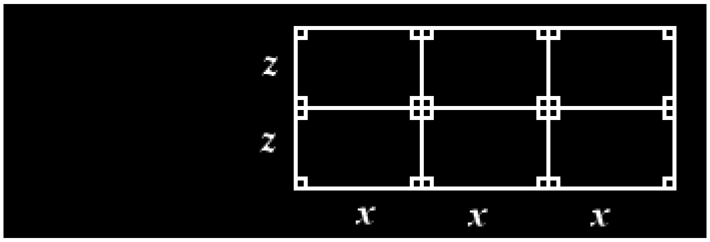
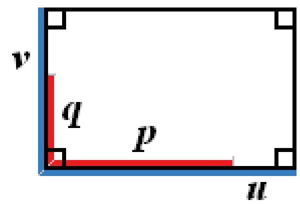
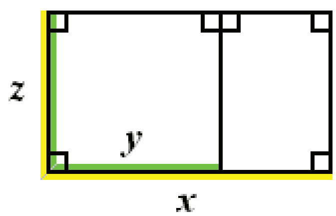
and nz. In the drawing below, $m = 3$ and $n = 2$.



3. Combining the previous two steps with the Archimedean Property of real numbers to show that if a rectangle exists, then there is a rectangle whose sides have dimensions u and v, where $u > p$ and $v > q$.



4. A trimming – down construction, which shows that if there is a rectangle whose sides have dimensions x and z and y is a positive number less than x, then there is a rectangle whose sides have dimensions y and z. Two applications of this combine with the third step to prove Theorem 8.



The proofs for several of these steps are quite lengthy in their own right. Therefore we shall now move forward, with the details in an Appendix to this section. ■

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The All – or – Nothing Theorem for angle sums

Lecture 16 begins here.

The preceding result on rectangles has an immediate consequence for angle sums of triangles.

Theorem 9. *If a rectangle exists in a neutral plane $\mathbb{P}$, then every right triangle in $\mathbb{P}$ has an angle sum equal to* $180°$.

Proof. Suppose we are given right triangle $\triangle ABC$ with a right angle at B. By the preceding result there is a rectangle $\square WXYZ$ such that $|AB| = |WX|$ and $|BC| = |XY|$. By S.A.S. we have $\triangle ABC \cong \triangle WXY$; in particular, the angle sums of these triangles are equal. On the other hand, the proof of Theorem 7 implies

Go to lecture16a.pdf for the rest of the material!