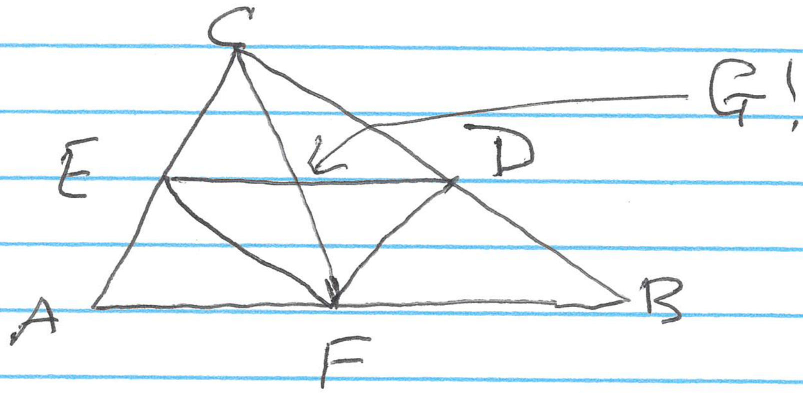
SOLUTION TO ORIGINAL PROB. 4



Draw CF.

Since $F \in \text{Int}\,\angle ACB$, $(CF$ meets DE because $D \in (BC)$ and $E \in (AC)$ so that $\angle ACB = \angle DCE$.

Let G be the intersection. CLAIM $G \in (DE)$

Betweenness conditions $\Rightarrow$ A, E on same side CF

B, D on same side CF, A, B on opp sides. Hence (DE) meets CF, and this can only happen at G.

Apply additivity of δ

$\delta\Delta AFC + \delta\Delta FCB = \delta\Delta ABC$. LHS =

$\cancel{\delta\Delta ABC}$

$(\delta\Delta AEF + \delta\Delta EFC) + (\delta\Delta BDF + \delta\Delta DFC) =$

$(\delta\Delta AEF + \delta\Delta EFG + \delta\Delta EGC) +$

$(\delta\Delta BDF + \delta\Delta DFG + \delta\Delta DGC)$.

Now all δ's are positive and

$$\delta\Delta DEF = \delta\Delta EFG + \delta\Delta DFG.$$

Therefore the expression at the bottom of page 1 implies

$$\delta\Delta ABC = \delta\Delta DEF + \delta$$

where $\delta > 0$ is a sum of other triangle defects. Therefore $\delta\Delta DEF < \delta\Delta ABC$.

Grading Scheme

5 Draw CF
8 Show (DE) meets (CF)
5 Know additivity property of δ
10 Apply additivity repeatedly to get

$$\delta \triangle ABC = \delta \triangle DEF + \text{positive}$$

5 Finish correctly.