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Quiz 1, Spring 2022

Let L be a line in a plane satisfying the Incidence Axioms and the Ruler Postulate, let A and B be two distinct points on L, and let f be a real valued ruler function on L such that $f(A) < f(B)$.

(1) State a numerical inequality such that X lies on the ray $[BA$ if and only if $f(X)$ satisfies this inequality.

(2) Let $C \in L$ be such that $A * B * C$ holds. State a numerical inequality such that X lies on the ray $[BC$ if and only if $f(X)$ satisfies this inequality.

SOLUTIONS

(1) The quickest way to answer this is to consider the points of BA which do not lie on $[BA$. These are the points for which $X * A * B$ is false. Now we know that $X * A * B$ is false if and only if both $f(A) < f(B) < f(X)$ and $f(X) < f(B) < f(A)$ are false. Since $f(A) < f(B)$ was assumed, the condition for $X \notin [BA$ reduces to the property that $f(A) < f(B) < f(X)$ is false. Therefore $X \in [BA$ is true if and only if $f(X) \leq f(B)$.■

(2) Since $A * B * C$ holds if and only if $f(A) < f(B) < f(C)$ or $f(C) < f(B) < f(A)$ and we know that $f(A) < f(B)$, it follows that $f(B) < f(C)$. As in (1), a point X does not lie on $[BC$ if and only if $X * B * C$ is false; in other words, this happens if and only if $f(X) < f(B)$ is false since $f(B) < f(C)$. Therefore $X \in [BC$ is true if and only if $f(X) \geq f(B)$.■