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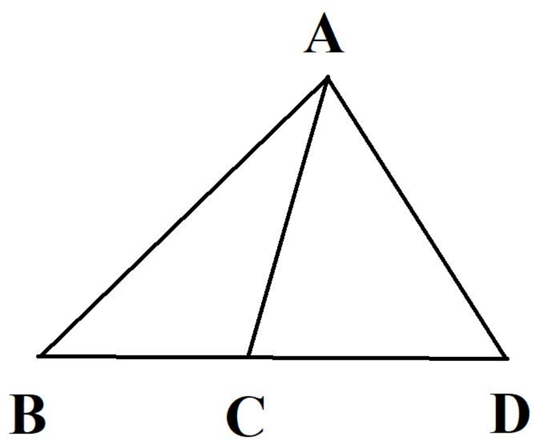
Math 133, Fall 2018, Quiz 2

Let B, C, D be points on a line L such that $B * C * D$, and let A be a point which is not on L. Prove that the three distances $|AB|$, $|AC|$ and $|AD|$ cannot be equal. [Hint: One can use the Exterior Angle and Isosceles Triangle Theorems.]



Solution. We shall assume that the three distances are equal and derive a contradiction. Under this assumption we know that we have three isosceles triangles ΔABC, ΔABD and ΔADC. By the Isosceles Triangle Theorem we have the following equalities:

$$|\angle ABC = \angle ABD| \quad = \quad |\angle ACB| \quad = \quad |\angle ADB = \angle ADC| \quad = \quad |\angle ACD|$$

Call this common value θ. Since $\angle ACB$ and $\angle ACD|$ are supplementary (because $B*C*D$), it follows that

$$180^\circ \quad = \quad |\angle ACB| \; + \; |\angle ACD| \quad = \quad 2\,\theta$$

so that $\theta = 90^\circ$. On the other hand, the Exterior Angle Theorem implies that

$$\theta \quad = \quad |\angle ACB| \quad > \quad |\angle ADC| \quad = \quad \theta$$

which is a contradiction. The source of the contradiction is the assumption that $|AB|$, $|AC|$ and $|AD|$ are all equal, so this must be false, and therefore the conclusion in the problem must be true.■