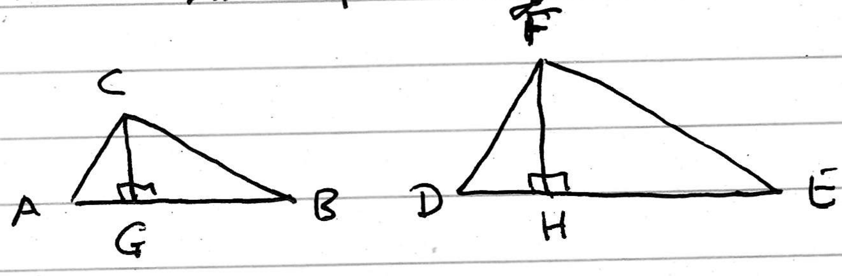
Quiz 3A

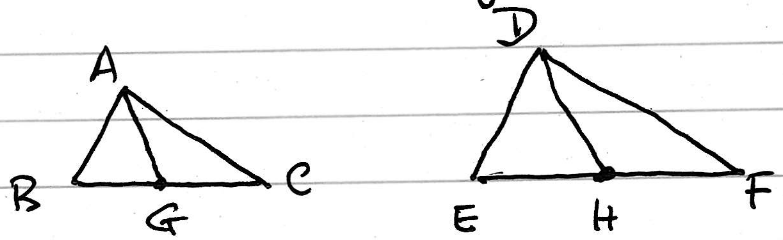
Suppose we are given $\Delta ABC \sim \Delta DEF$ in the Euclidean plane with ratio of similitude k, and for the sake of convenience assume that $|\measuredangle ACB| = |\measuredangle DFE| = 90$, so that the altitudes from C and F meet AB and DE in points $G \in (AB)$ and $H \in (DE)$ respectively.



Prove that $d(F,H) = k \cdot d(C,G)$. [HINT: What can we say about ΔAGC and ΔDHF?].

Quiz 3B

Suppose we are given $\Delta ABC \sim \Delta DEF$ in the Euclidean plane with ratio of similitude k, and let G and H denote the midpoints of $[BC]$ and $[EF]$ respectively.



Prove that $d(D,H) = k\, d(A,G)$. [<u>Hint</u>: What can we say about ΔABG and ΔDEH?].

SOLUTIONS FOR QUIZ 3

$\boxed{\text{A.}}$ We are given that $\Delta ABC \sim_k \Delta DEF$, so

$$|\measuredangle CAG = \measuredangle CAB| = |\measuredangle FDH = \measuredangle FDE|.$$

Also, $|\measuredangle CGA| = |\measuredangle FHD| = 90$ is given.
Therefore $\Delta CAG \sim \Delta FDH$ by SS similarity.
The ratio of similitude is equal to

$\dfrac{d(F,D)}{d(C,A)} = k$ (by hypothesis), so we also have

$\dfrac{d(F,H)}{d(C,G)} = k$, which means $d(F,H) = k\,d(C,G)$. $\square$

$\boxed{\text{B}}$ We are given $\Delta ABC \sim_k \Delta DEF$, so

$|\measuredangle ABC = \measuredangle ABG| = |\measuredangle DEH = \measuredangle DEF|$, and

$$\frac{d(D,E)}{d(A,B)} = k, \quad \frac{d(E,F)}{d(B,C)} = k.$$

Since $d(E,H) = \frac{1}{2} d(E,F)$ and

$d(B,G) = \frac{1}{2} d(B,C)$ we also have

$$\frac{d(E,H)}{d(B,G)} = \frac{\frac{1}{2}\,d(E,F)}{\frac{1}{2}\,d(B,C)} = \frac{d(E,F)}{d(B,C)} = k$$

Therefore $\Delta ABG \sim_k \Delta DEH$ by SAS similarity,

so that $\dfrac{d(D,H)}{d(A,G)} = k$, or $d(D,H) = k \cdot d(A,G)$. $\square$