Practice Problems

1. Suppose we are given ΔABC with $|\measuredangle ABC| = 90°$. Let $H \in (AC)$ be the foot of the perpendicular from B, and let $D \in (BC)$ be such that $[AD$ bisects $\measuredangle BAC$.

   Prove that (AD) and (BH) have a point in common. (NEUTRAL PLANE).

2. Aristotle's Theorem states that the set of all points X in the coordinate plane such that $2|AX| = |BX|$ is a circle. If $A = (0,0)$ and $B = (3,0)$, find the ~~equation~~ center of that circle.

3. Let ΔABC lie in a hyperbolic plane, with $D \in (BC)$, $E \in (AC)$, $F \in (AB)$. Prove that the angular defects satisfy

   $$\delta \Delta DEF < \delta \Delta ABC.$$

   Hint: Cut ΔABC into two pieces along $[CF]$. ~~where~~ Why do (CF) and (DE) have a point in common? This splits ΔACF and ΔBCF into 3 triangles each. How are $\delta\Delta ABC$, $\delta\Delta ACF$, $\delta\Delta BCF$ related? What else can we say?

4. Suppose a, b, c, d, e, f are positive real numbers such that $\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$.

Prove that $\frac{a+b+c}{d+e+f} = \frac{a}{d}$.

Suppose $\triangle ABC$ and $\triangle DEF$ are in a Euclidean plane and $\triangle ABC \sim \triangle DEF$ with ratio of similitude r. What is the ratio of these triangles' perimeters?

5. If $\triangle ABC$ lies in a neutral plane, prove that there is no line L containing points from each of (AB), (AC) and (BC).

6. For each of the following, determine if it is true in
(1) every neutral plane
(2) only Euclidean planes
(3) only hyperbolic planes
(4) no neutral planes.

(a) In $\triangle ABC$ we have $|AB| < |AC| + |BC|$.
(b) If $|\measuredangle ABC|$ is isosceles and $|\measuredangle ABC| = 90°$, then $|\measuredangle BAC| < 45°$.
(c) ——— ditto ———, then $|\measuredangle BAC| = 45°$
(d) ——— ditto ———, then $|\measuredangle BAC| > 45°$.
(e) The AAS congruence thm. for triangles.
(f) All perpendiculars to line L meet at some $X \notin L$.
(g) There are lines L, M, N so $L \parallel M$, $N \parallel M$ but $L \not\parallel N$.