

## Practice Problems

1. Suppose we are given  $\triangle ABC$  with  $\angle ABC = 90^\circ$ . Let  $H \in (AC)$  be the foot of the perpendicular from  $B$ , and let  $D \in (BC)$  be such that  $[AD]$  bisects  $\angle BAC$ .  
(~~NEUTRAL PLANE~~)  
Prove that  $(AD)$  and  $(BH)$  have a point in common. (NEUTRAL PLANE).

2. Aristotle's Theorem states that the set of all points  $X$  in the coordinate plane such that  $2|AX| = |BX|$  is a circle. If  $A = (0, 0)$  and  $B = (3, 0)$ , find the ~~equation~~ center of that circle.

3. Let  $\triangle ABC$  lie in a hyperbolic plane, with  $D \in (BC)$ ,  $E \in (AC)$ ,  $F \in (AB)$ . Prove that the angular defects satisfy  $\delta \triangle DEF < \delta \triangle ABC$ .

Hint: Cut  $\triangle ABC$  into two pieces along  $[CF]$ . ~~where~~ Why do  $(CF)$  and  $(DE)$  have a point in common? This splits  $\triangle ACF$  and  $\triangle BCF$  into 3 triangles each. How are  $\delta \triangle ABC$ ,  $\delta \triangle ACF$ ,  $\delta \triangle BCF$  related? What else can we say?

4. Suppose  $a, b, c, d, e, f$  are positive real numbers such that  $\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$ .

Prove that  $\frac{a+b+c}{d+e+f} = \frac{a}{d}$ .

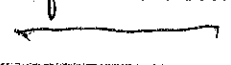

Suppose  $\triangle ABC$  and  $\triangle DEF$  are in a Euclidean plane and  $\triangle ABC \sim \triangle DEF$  with ratio of similitude  $r$ . What is the ratio of these triangles' perimeters?

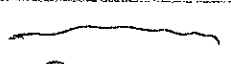

5. (a) If  $\triangle ABC$  lies in a neutral plane, prove that there is no line  $L$  containing points from each of  $(AB)$ ,  $(AC)$  and  $(BC)$ .

(b) For each of the following, determine if it is true in  
 (1) every neutral plane  
 (2) only Euclidean planes  
 (3) only hyperbolic planes  
 (4) no neutral planes.

(a) In  $\triangle ABC$  we have  $|AB| < |AC| + |BC|$ .

(b) If  $\angle ABC$  is isosceles and  $\angle ABC = 90^\circ$ , then  $|\angle BAC| < 45^\circ$ .

(c)  ditto , then  $|\angle BAC| = 45^\circ$

(d)  ditto , then  $|\angle BAC| > 45^\circ$ .

(e) The AAS congruence thm. for triangles.

(f) All perpendiculars to line  $L$  meet at some  $X \notin L$ .

(g) There are lines  $L, M, N$  so  $L \parallel M, N \parallel M$  but  $L \not\parallel N$ .