Exercises

1. Suppose $x, y, z \in L$ are distinct. Prove that $L = xy = xz = yz$. [Useful identity in more substantial proofs.]

3. More complicated identities like #1 $\quad A \neq B$
   (a) If $X \in (AB$, then $(AX = (AB$.
   (b) If $C * A * B$, then $(AC = (AB^{op}$.

   Hint for (b): If $f : AB \to \mathbb{R}$ ruler fcn. with $f(B) > f(A)$, then $-f$ satisfies $-f(C) > -f(A)$.

2. If $A \neq B$, prove that $(AB^{op}$ consists of all X such that $X * A * B$.

4. Verify the following using ruler functions

$$\text{If } X \neq Y \text{ in } \left\{\begin{matrix} [AB] \\ (AB) \\ [AB \end{matrix}\right\} \text{ and } X * Z * Y, \text{ then also}$$

$$Z \in \left\{\begin{matrix} [AB] \\ (AB) \\ [AB \end{matrix}\right\}.$$