

Number systems

What we need for the course.

- \mathbb{N} nonnegative integers
- \mathbb{Z} integers
- \mathbb{Q} rationals
- \mathbb{R} reals MAIN SYSTEM
- \mathbb{C} complex numbers

Key property for the last three: (1) Can do $+$, $-$, \times , \div (if denominator nonzero).

(2) Ordering for first four \leq [reverse \geq]

Trichotomy, transitive

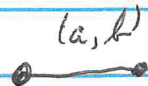
Positive elements closed under $+$ and \times .


(3) Reals — can approximate by rationals
largest such system.

If $x_1 \leq x_2 \leq x_3 \leq \dots \leq M$, then there

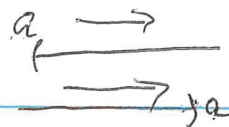
is a limit. Example: ∞ decimals $0.x_1x_2x_3x_4\dots$

(4) Complex numbers — expressible as
 $a + bi$, where a, b real, $i^2 = -1$.

Open interval (a, b) $a < x < b$ 

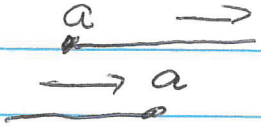
Closed interval $[a, b]$ $a \leq x \leq b$ 

Half open interval $[a, b)$ + $(a, b]$

Conventions (a, ∞) , $(-\infty, a)$ 

$$a < x$$

$$x < a$$

$[a, \infty)$, $(-\infty, a]$ 

Also, $\mathbb{R} = (-\infty, \infty)$. \longleftrightarrow

Systematic use of real numbers allows us to streamline much of the Greek (Euclid) approach to geometry.

Three parts of the course

- I. Use reasonable assumptions to coordinatize Euclidean geometry
- II. Indicate how vector methods are powerful tools for attacking many geometrical issues.
- III. Discuss + develop ^{classical} non-Euclidean geometry to a limited extent

In today's mathematics, the reasonable frame work for I is set theory

planes, lines etc. are sets
points are members of sets. $\text{lines} \subset \text{planes}$ etc.